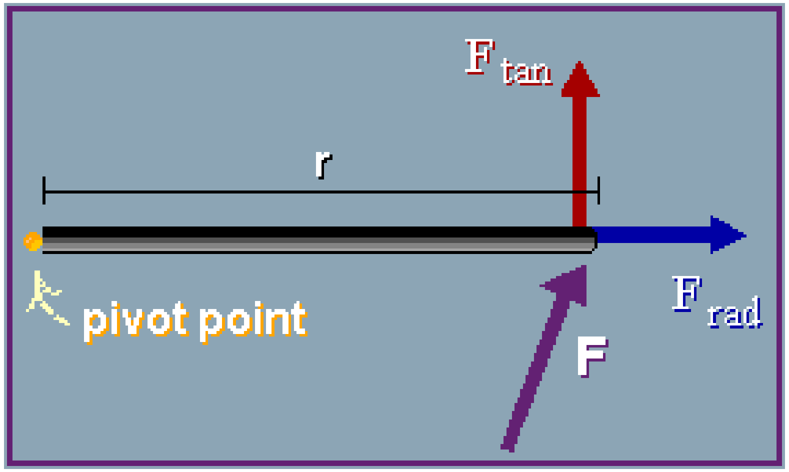
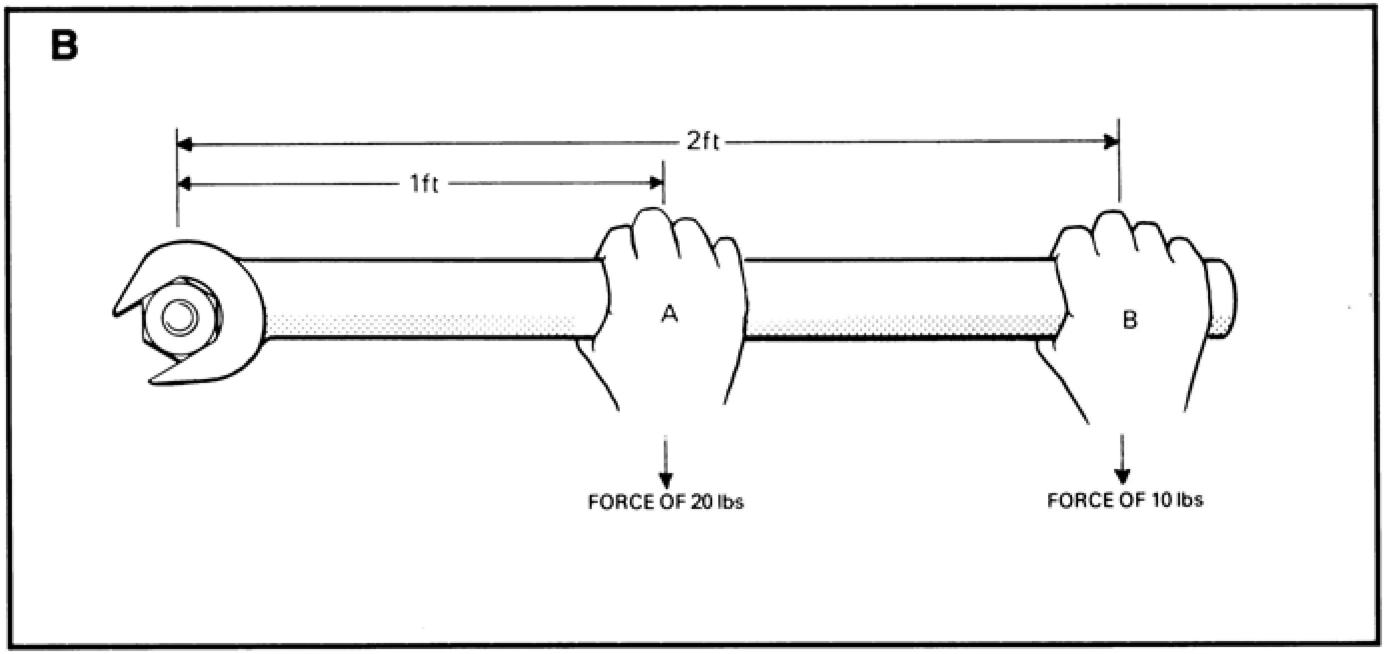

F tan
r
pivot point
F
F rad

Torque


B
2ft
1ft
A
B
FORCE OF 20 lbs
FORCE OF 10 lbs

Definition

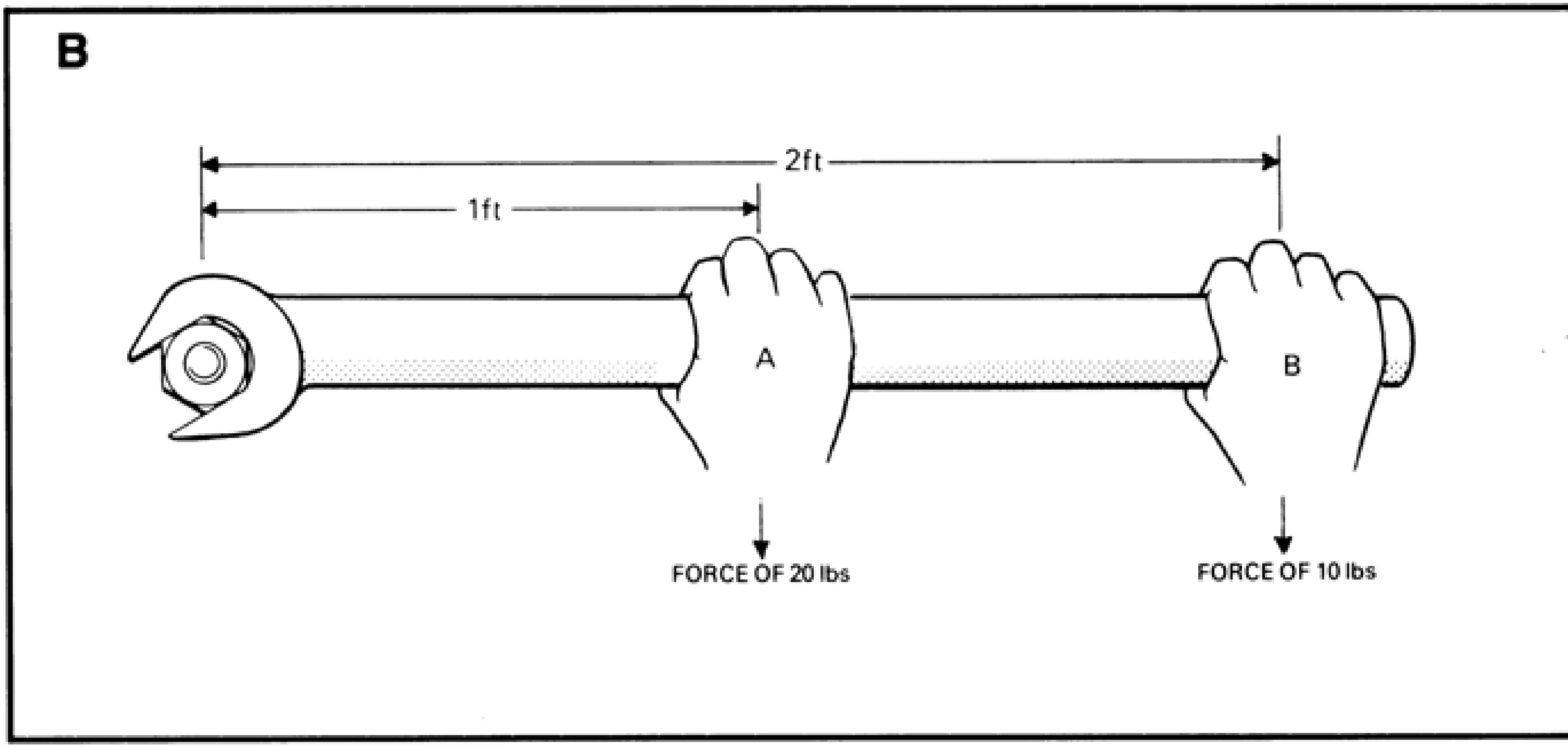
- **is a measure of how much a force acting on an object causes that object to rotate**, symbol is τ, (Greek letter tau)

- τ = $\mathbf{r}_\perp \mathbf{F}$ = $\mathbf{rF}\sin\theta$, r = distance from pivot to force, F is the applied force and θ is the angle between r and F

- SI unit of torque is N•m, this is **not the same as a joule**

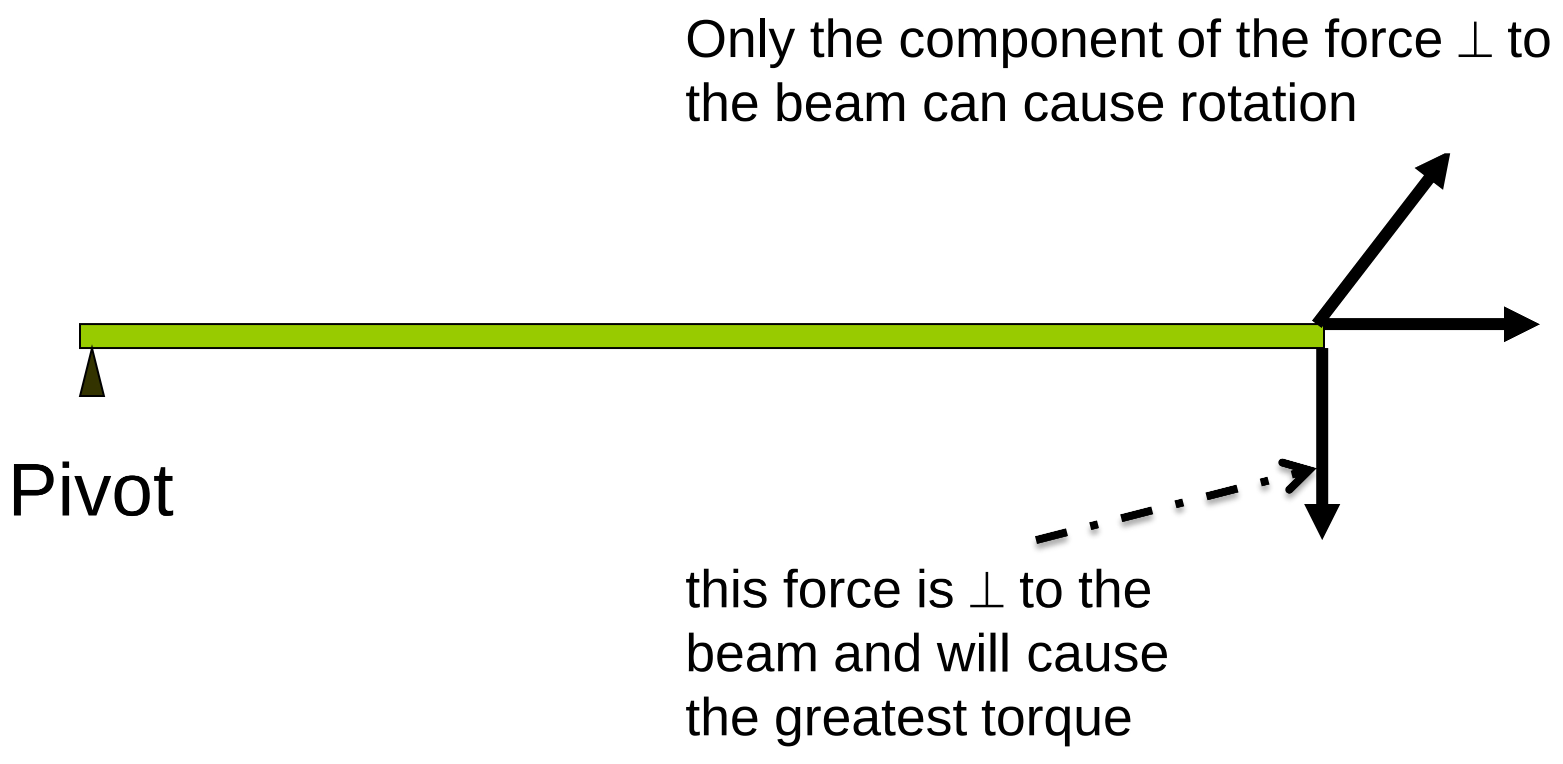
- Torque is a vector, $rF\sin\theta$ is a vector cross-product

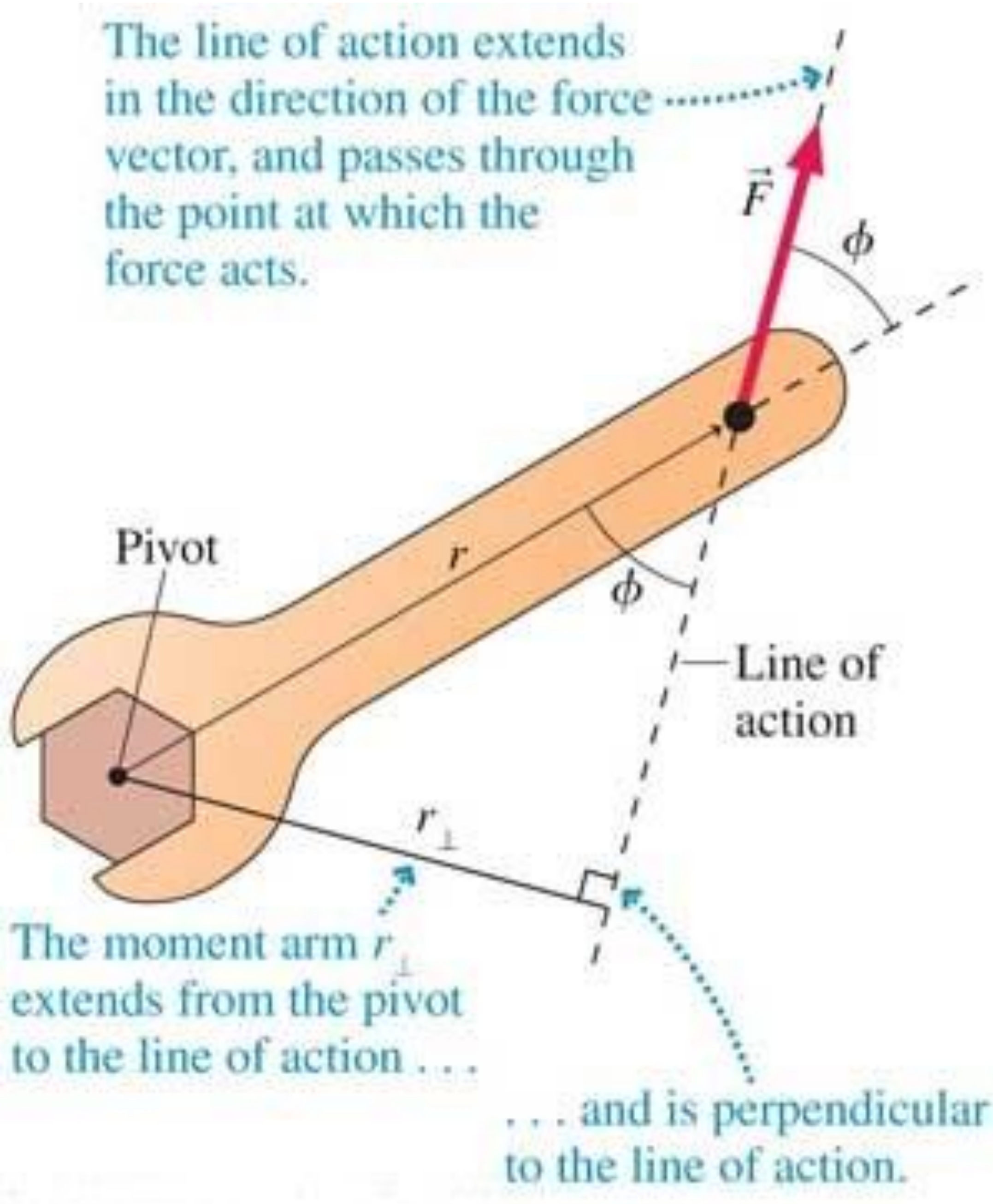
Equal Torques



$\tau = rF\sin\theta = 20$ lb•ft

- The magnitude of the torque depends on the magnitude of the applied force, the distance between the force and the pivot and the direction the force acts





The line of action extends in the direction of the force vector, and passes through the point at which the force acts.
$\vec{F}$
ϕ
Pivot
r
ϕ
Line of action
$r_\perp$
The moment arm $r_\perp$ extends from the pivot to the line of action . . .
. . . and is perpendicular to the line of action.

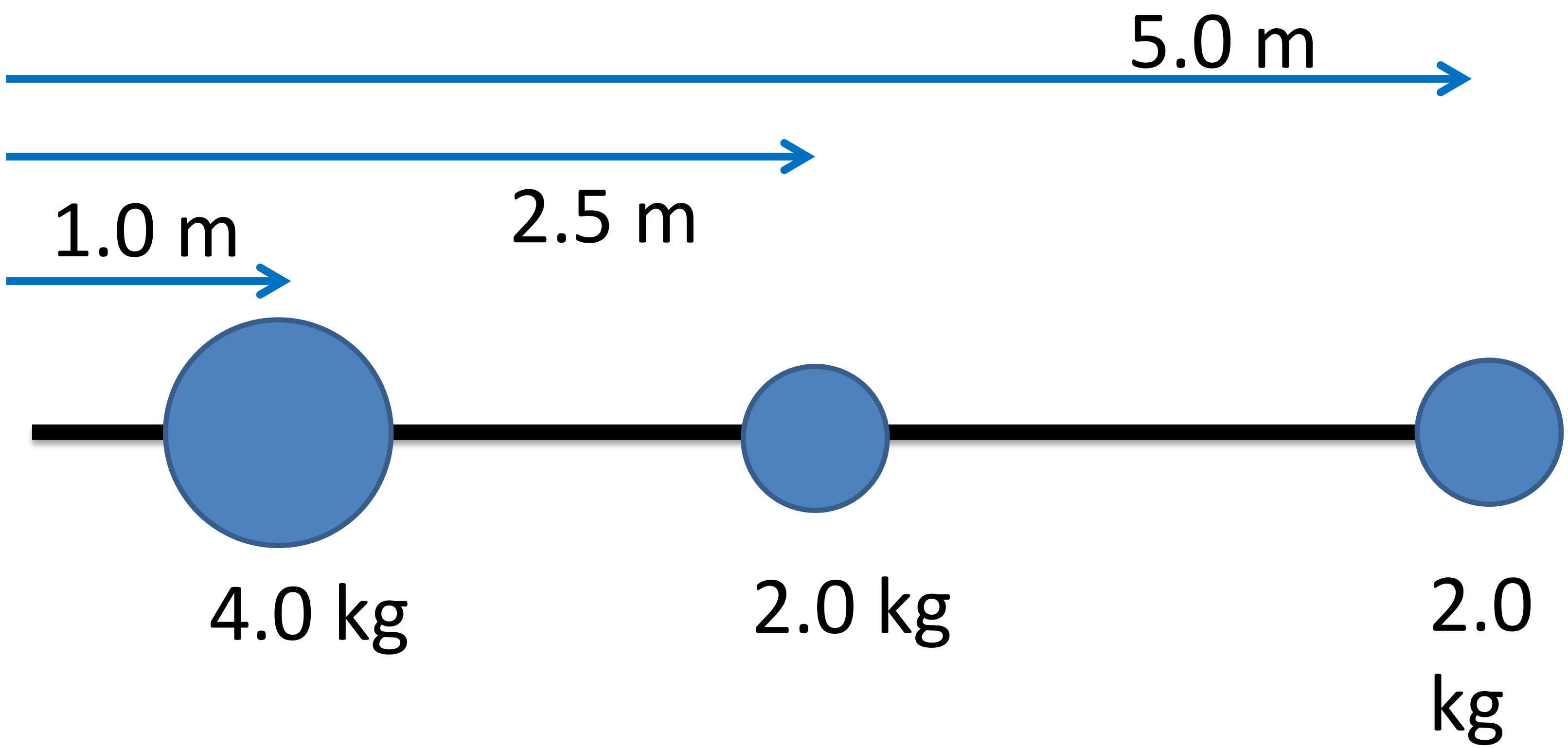
Centre of Mass (gravity)

- The point at which the mass of an object seems to act
- The centre of mass of a homogenous symmetric object is on an axis of symmetry
- An object is in a stable position when the centre of gravity is directly above or below the lowest support point.

- No torque will result when a vertical line passes through the centre of gravity and the point of support

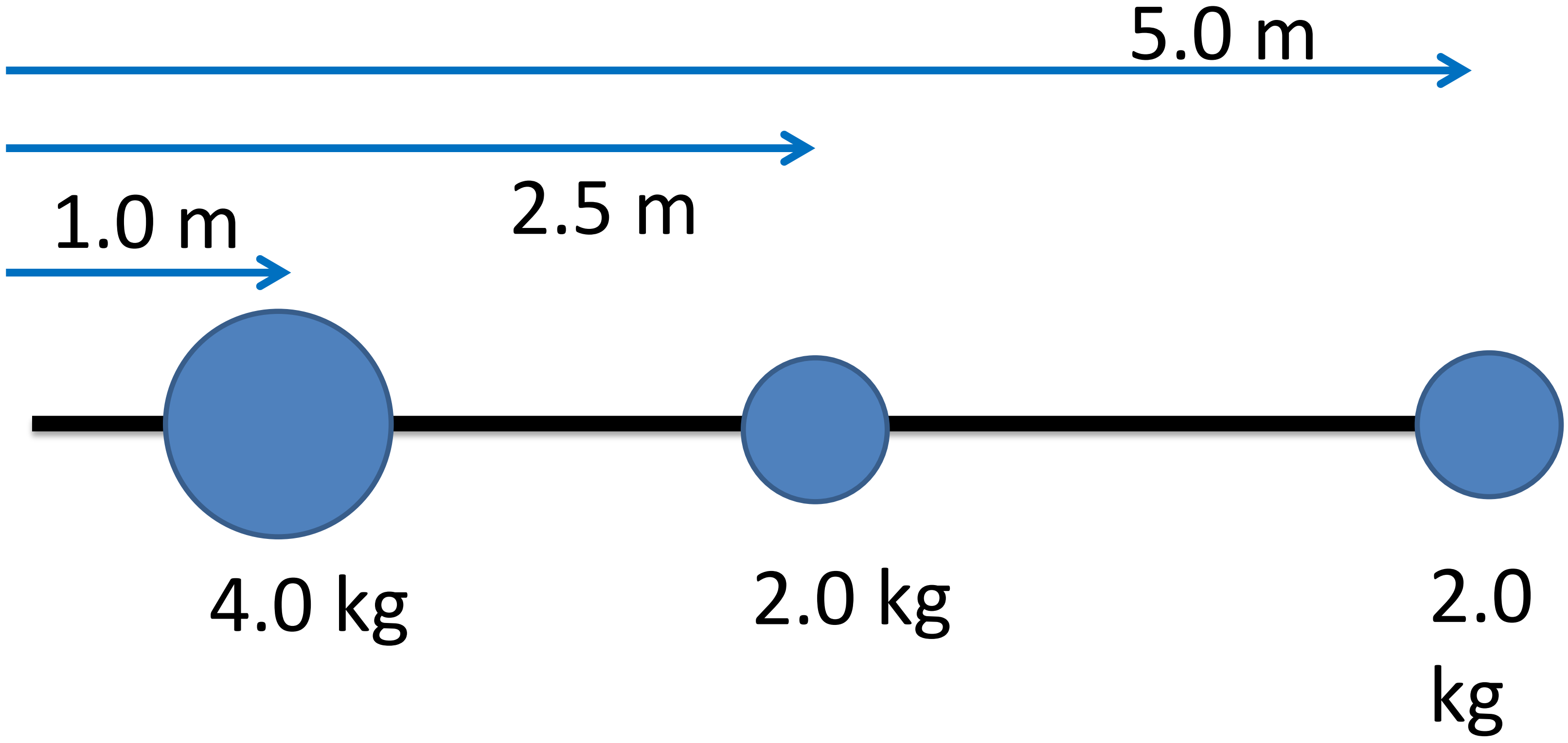
Locating Centre of Mass

- A long, thin beam has 3 masses distributed along it as shown. Locate the centre of mass relative to the left edge of the beam.

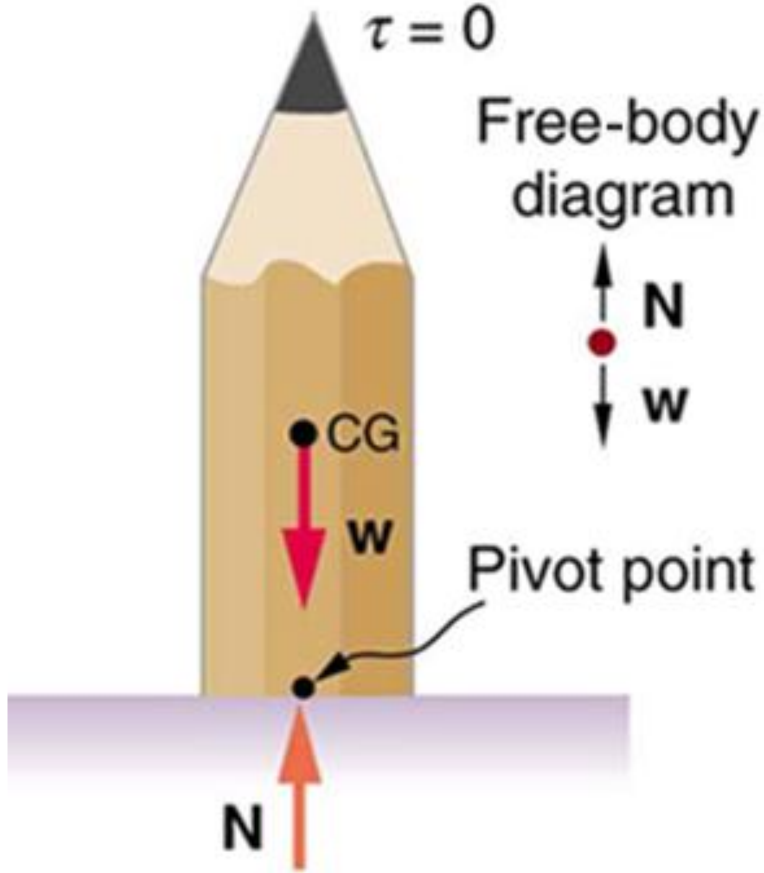


Solution

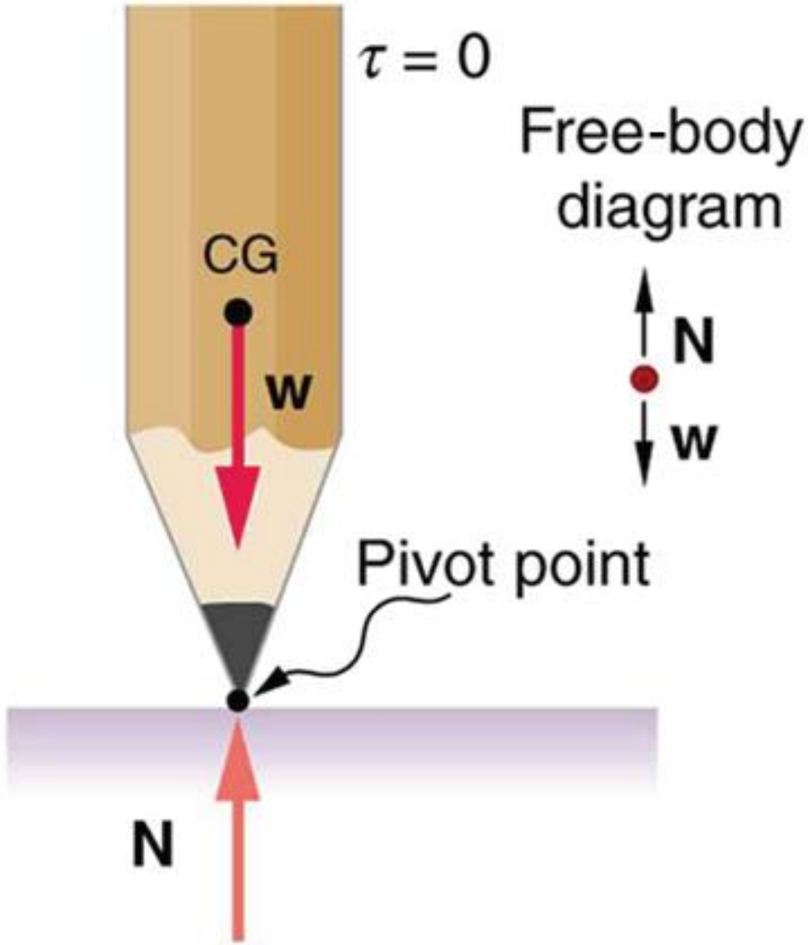
$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$



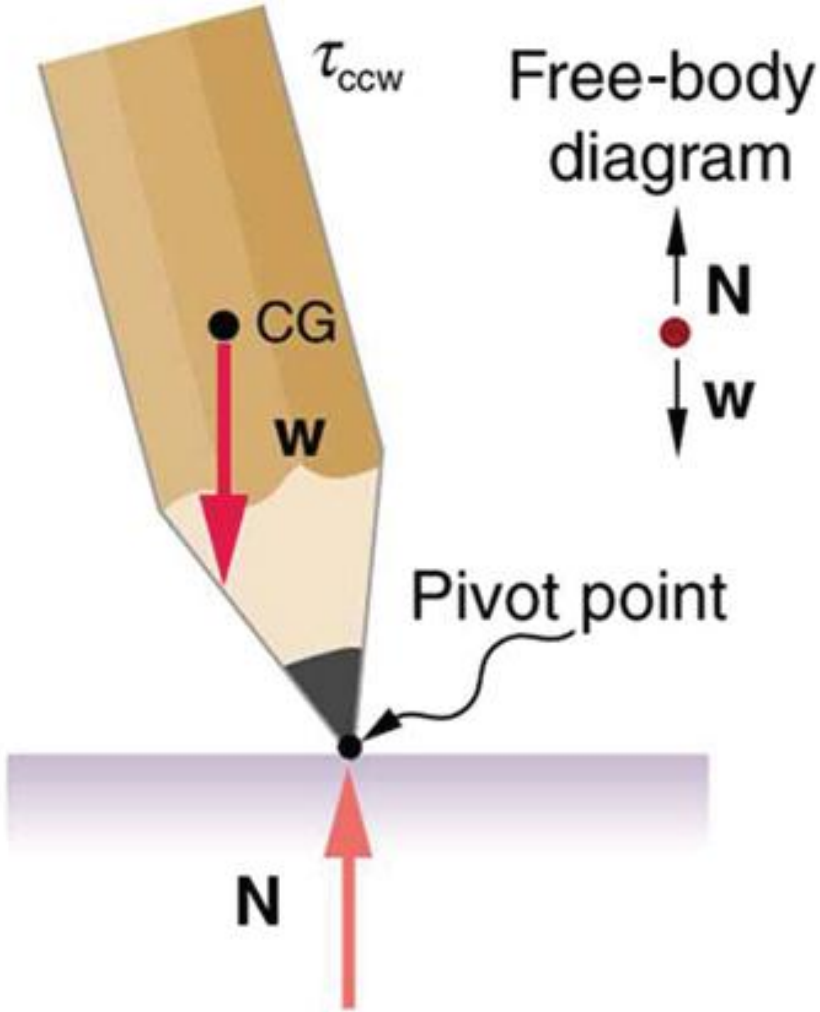
Net Torque = 0, stable

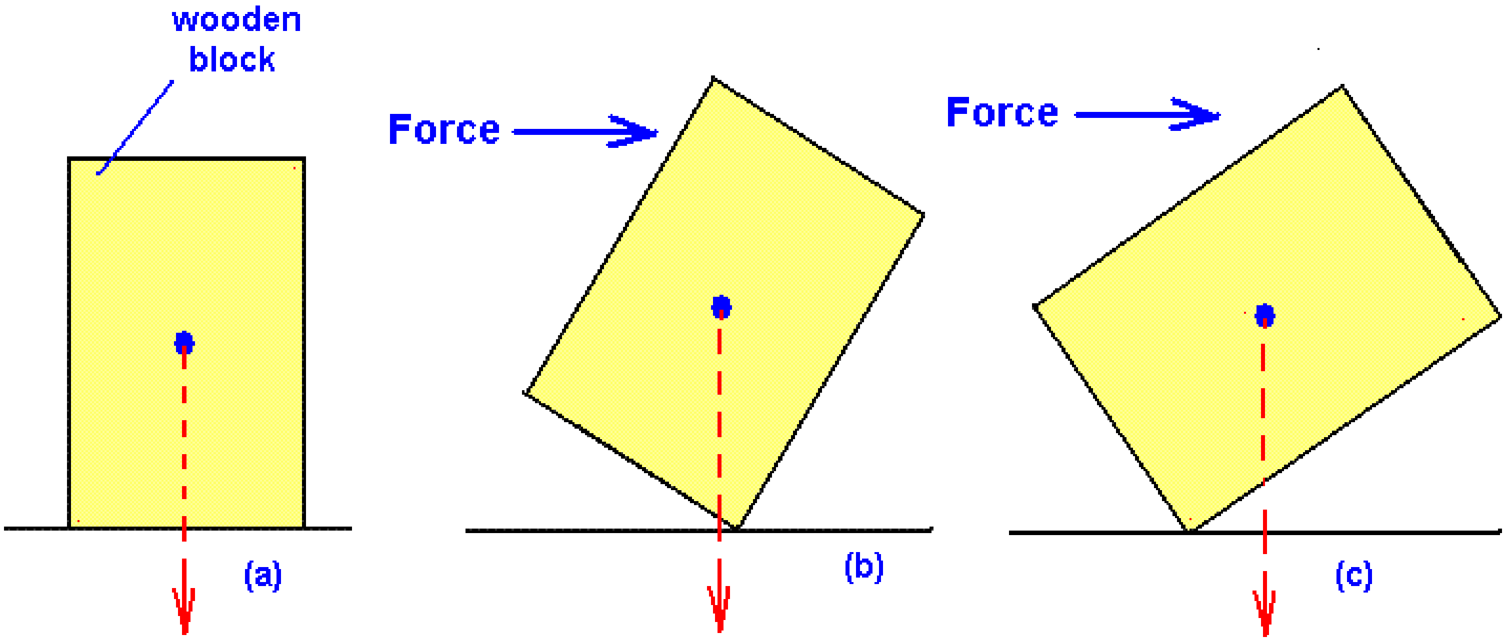


Net torque =0, unstable

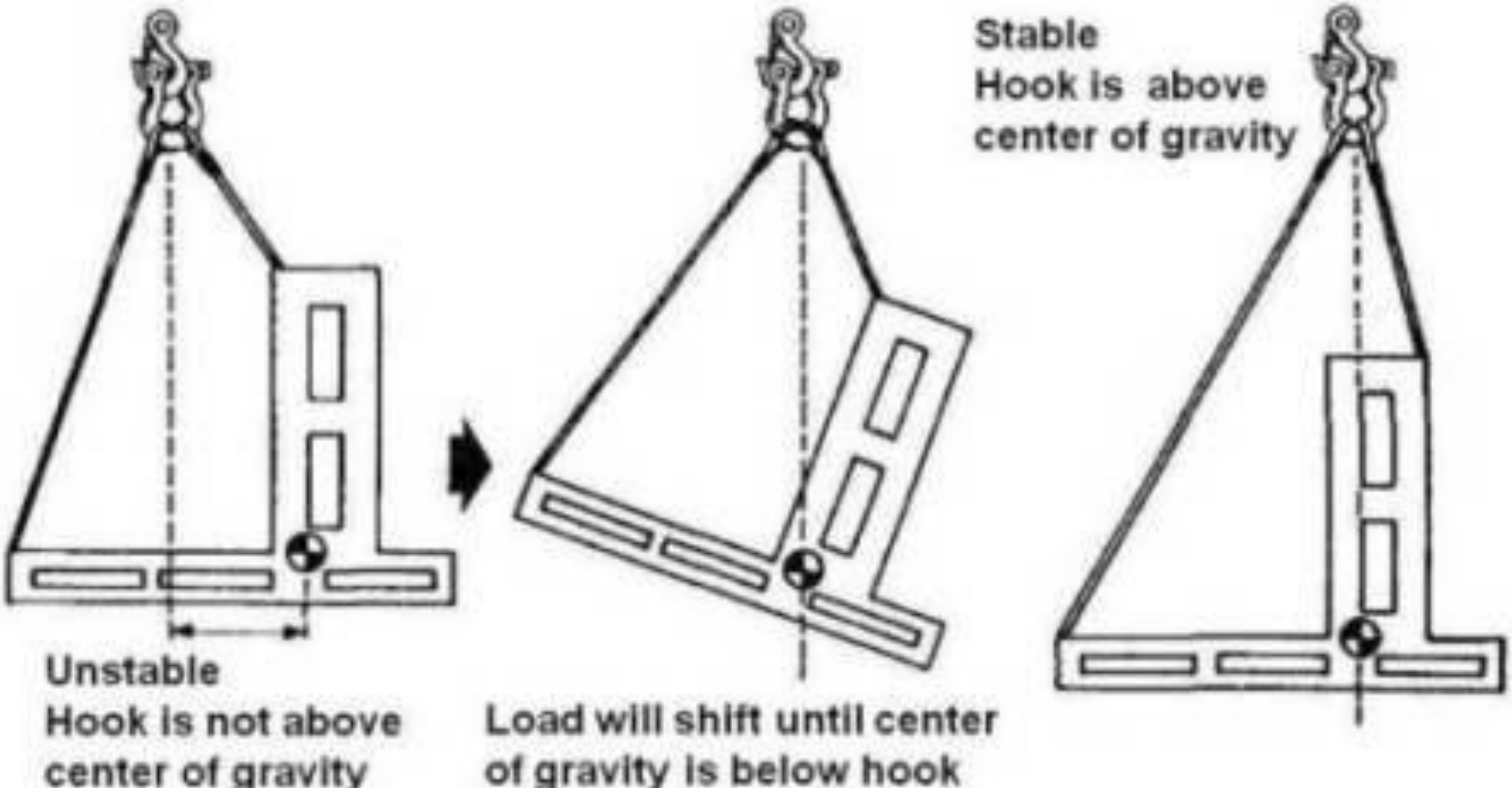


Net torque ≠ 0, Unstable





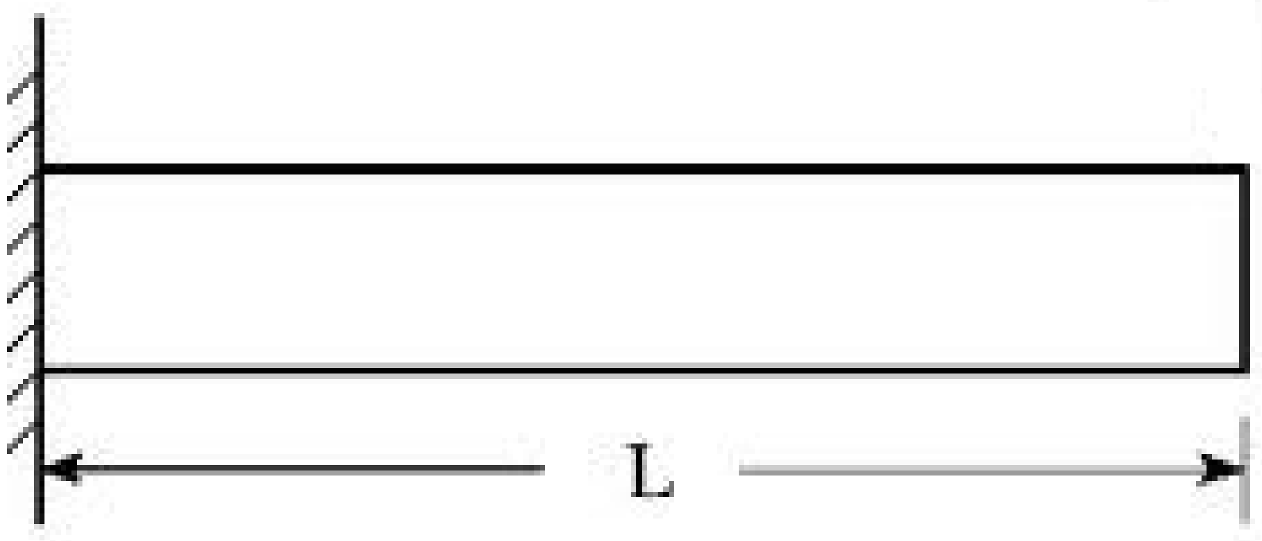
wooden block
Force
Force
(a)
(b)
(c)



Effect of Center of Gravity on Lift

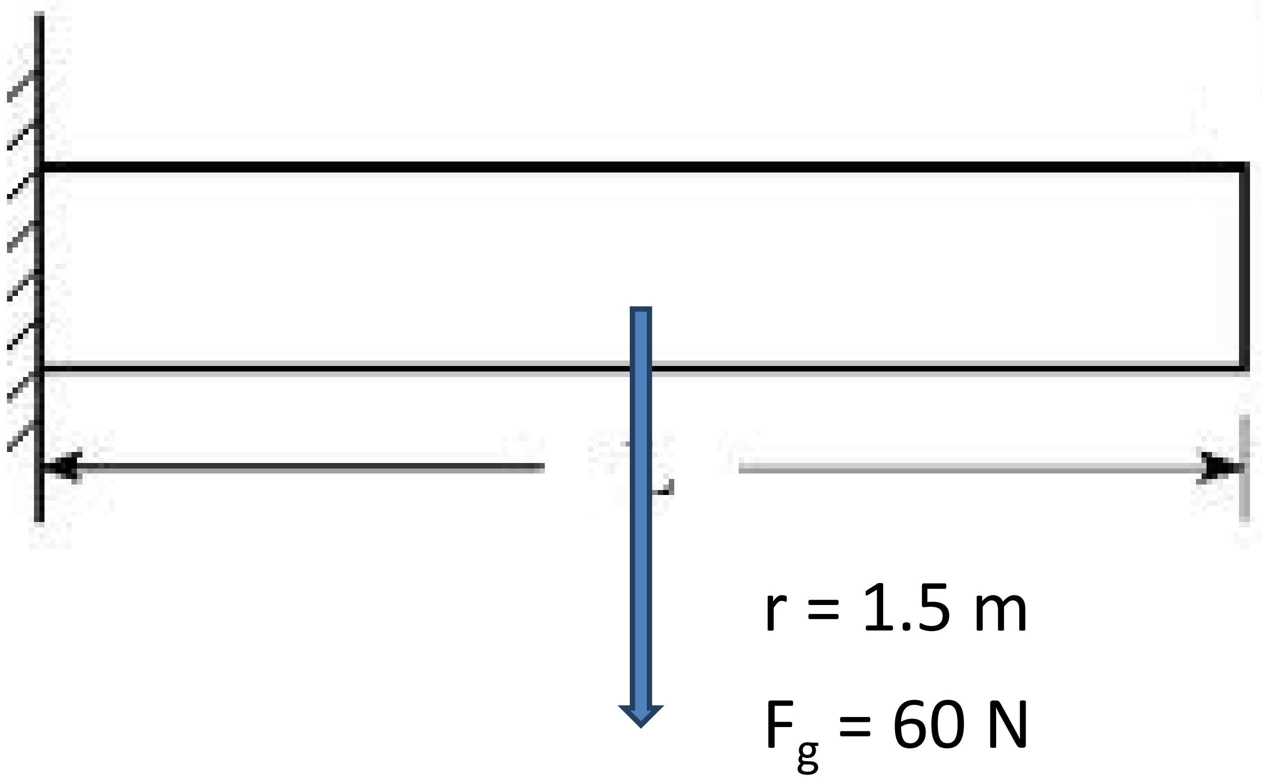
Example

- A 3.0 m long uniform beam has a weight of 60 N and is attached to a wall. Determine the torque acting on the attachment point.



Solution

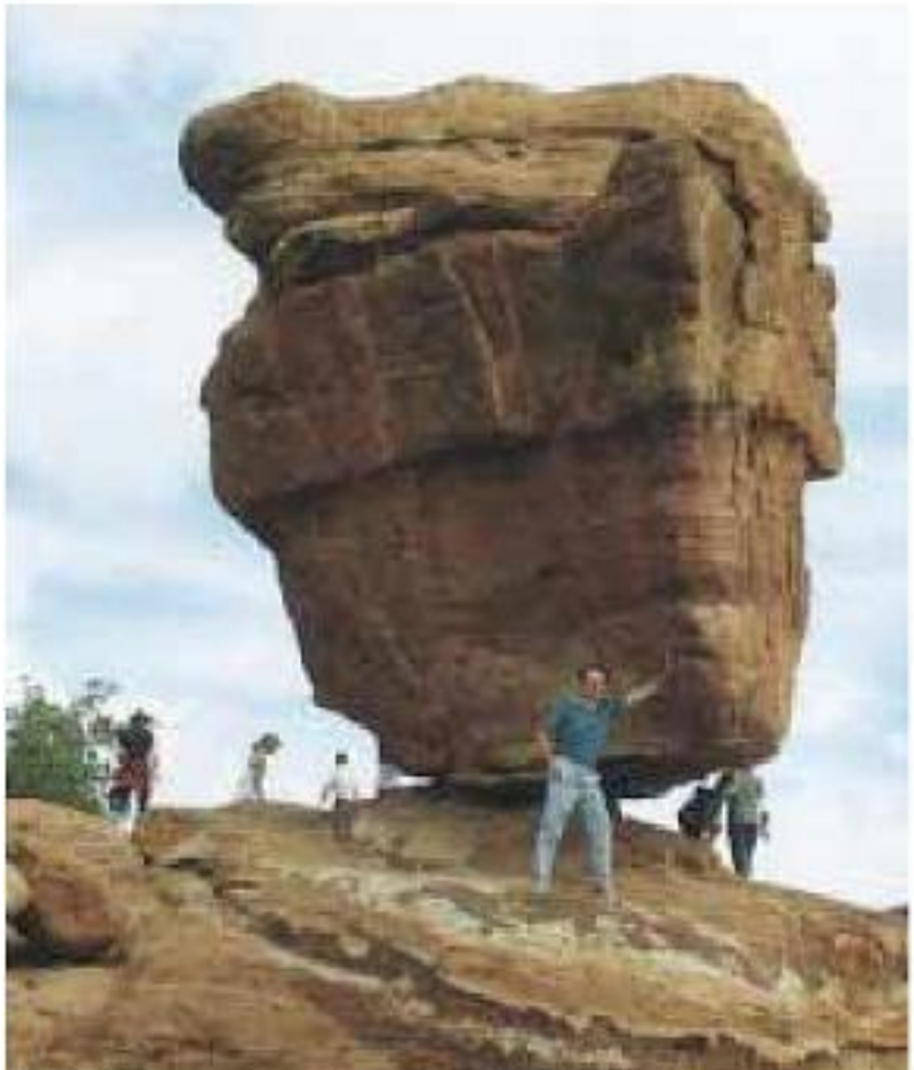
- The weight of the beam acts as if all the mass is concentrated at the centre of the beam.



Calculation

- $\tau = rF\sin\theta = 1.5 \text{ m} \times 60\text{N} \times \sin 90^o$

- $\tau = 90 \text{ N} \bullet \text{m}$



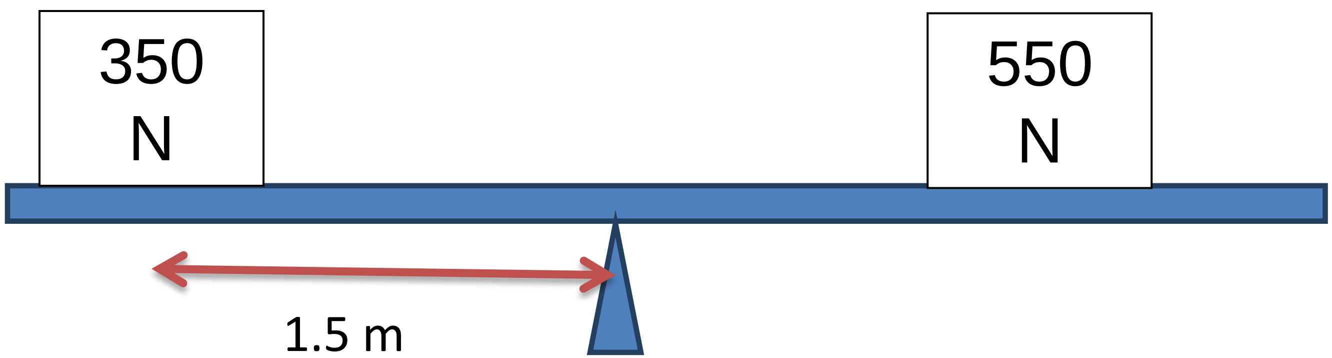
Static Equilibrium

- No acceleration and no rotation

- $\sum \mathbf{F} = 0$, $\sum \tau = 0$

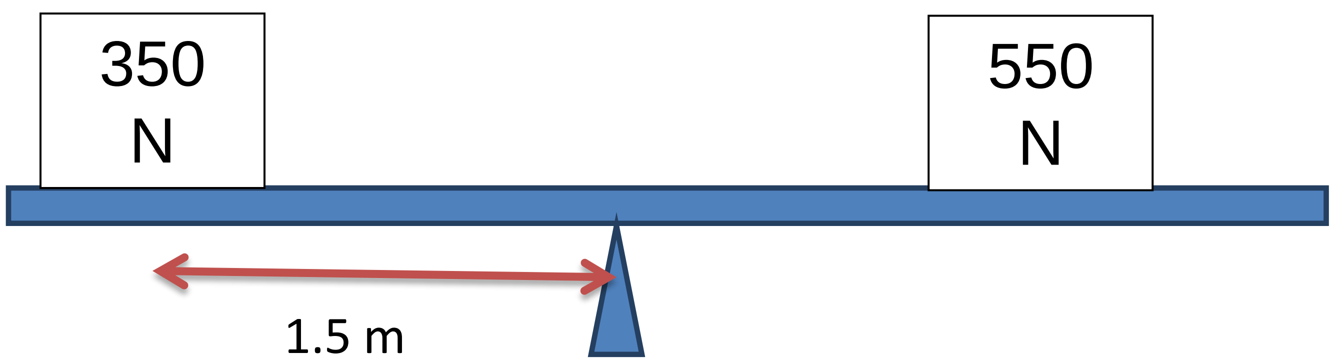
Example

- A uniform beam with a weight of 300 N is pivoted at its centre of gravity. A 350 N weight is placed 1.5 m from the pivot. How far from the pivot should a 550 N weight be placed so that the beam does not rotate?



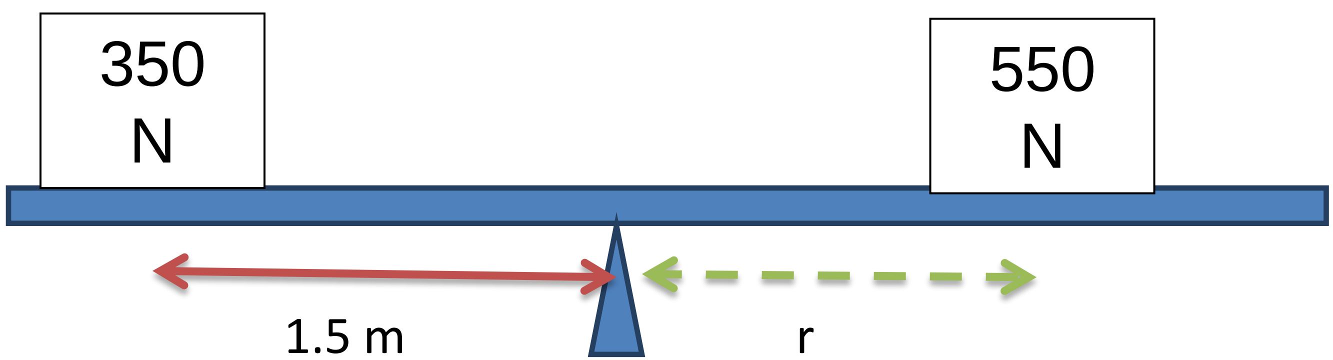
Solution

Since the beam is supported at the centre of gravity, the beam would be balanced without the placement of the extra weights.



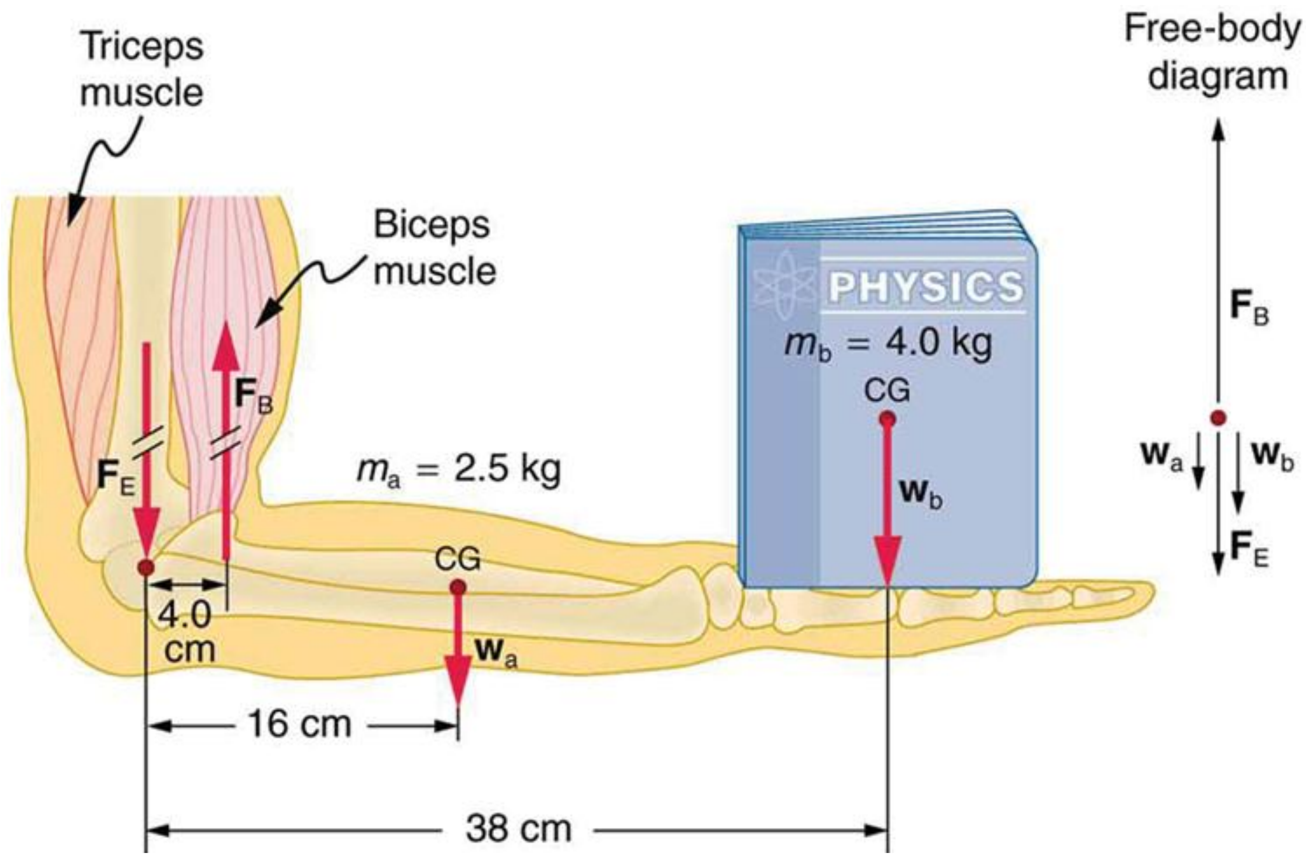
Solution

- $\tau_{350\text{ N}} + \tau_{550\text{ N}} = 0$ $\theta = 90^o$
- $\tau_{350\text{ N}} = - \tau_{550\text{ N}}$
- 1.5 m x 350 N = r x 550 N
- r = 0.95 m

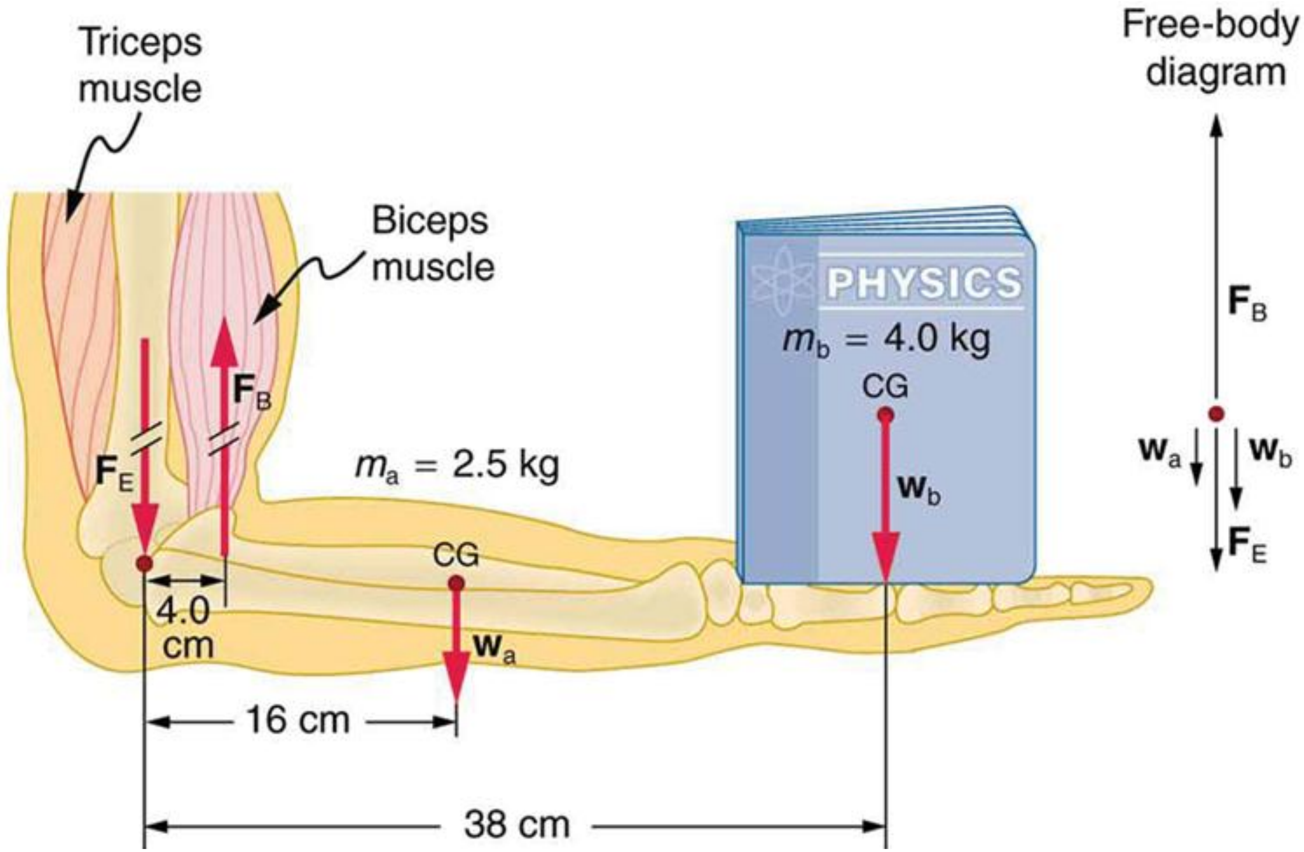


Example

- Determine the upward force exerted by the biceps muscle on the forearm.

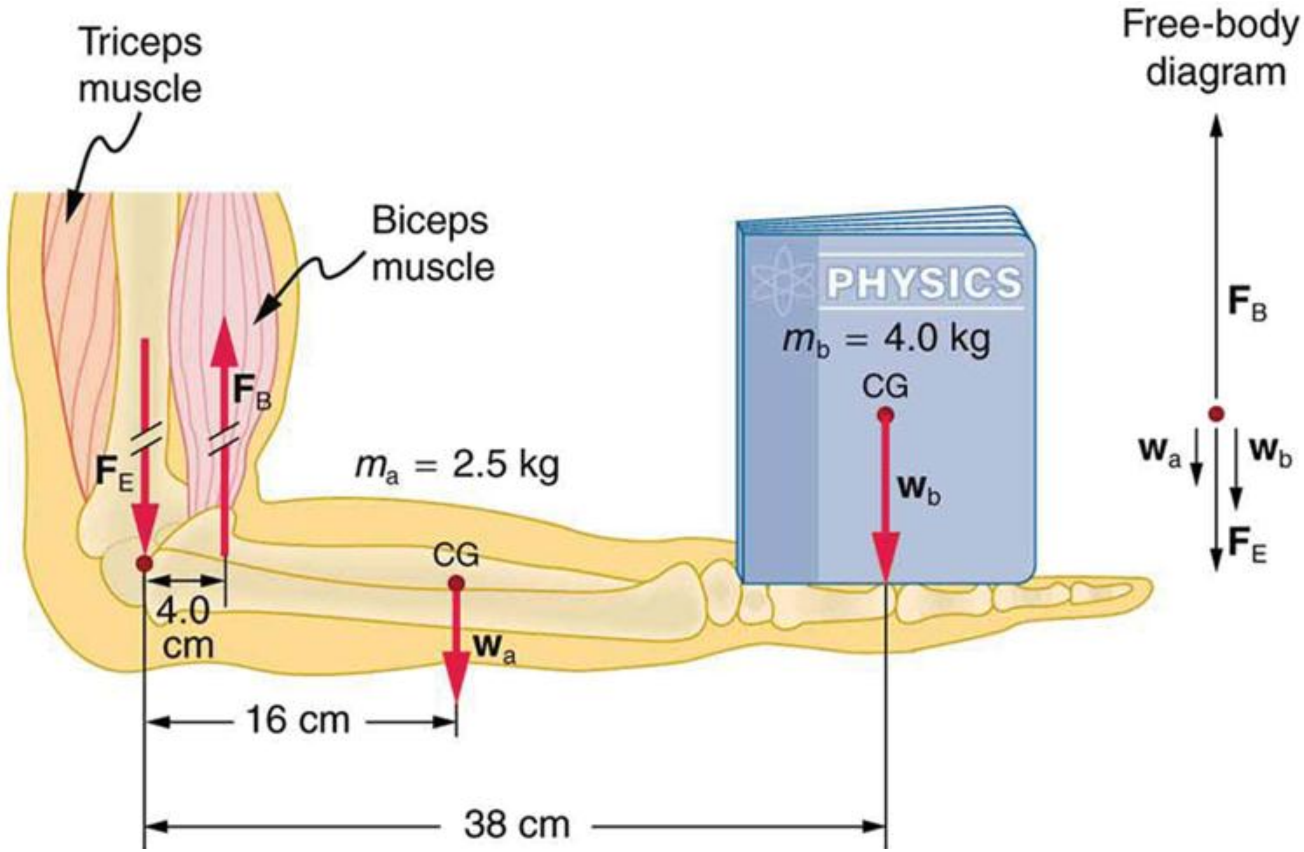


Solution



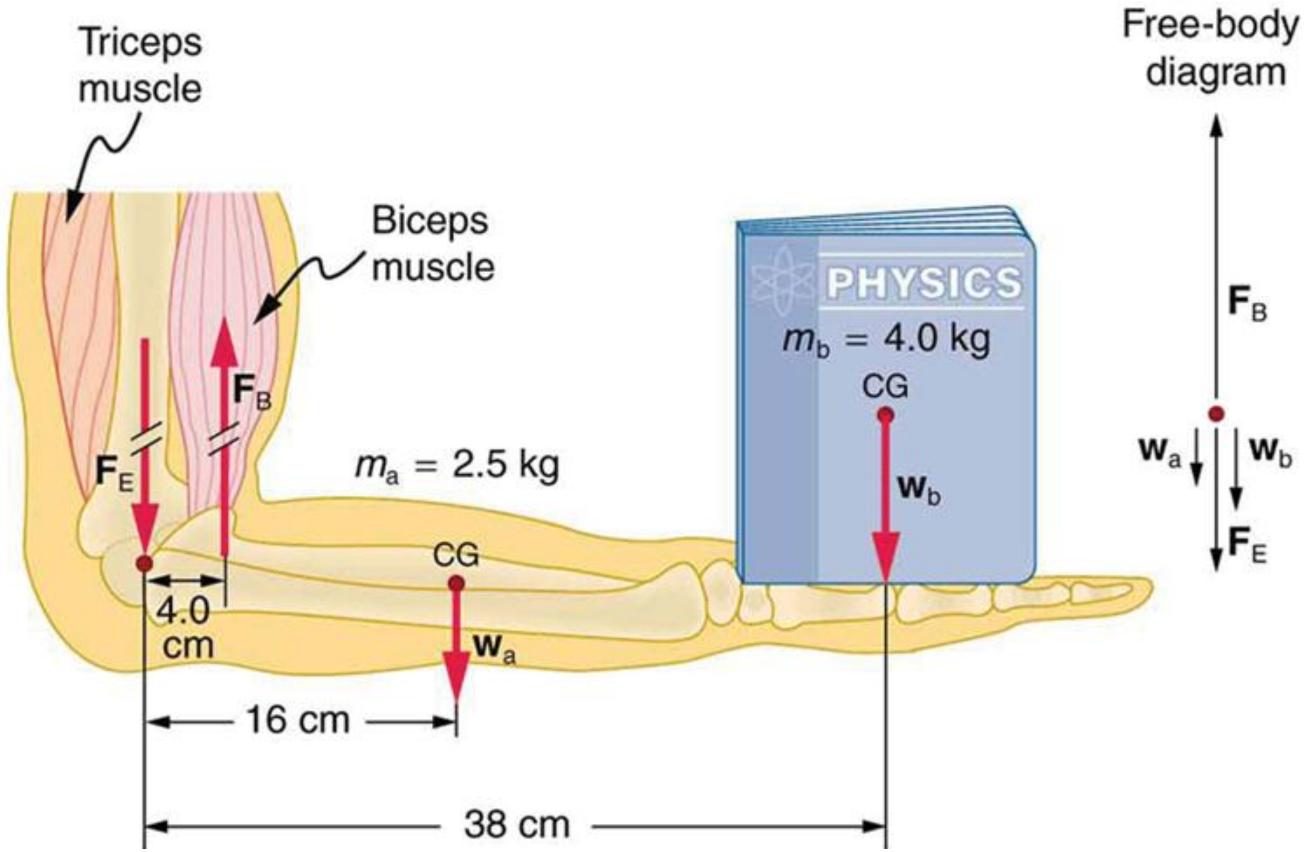
- $\tau_{book} + \tau_{arm} + \tau_{muscle} = 0, \quad \theta = 90^o$
- $-(40 \times 0.38) + -(25 \times 0.16) + \tau_{muscle} = 0$

Solution



- -15.2 N•m + -4.0 N•m + τ_{muscle} = 0
- τ_{muscle} = 19.2 N•m

Solution

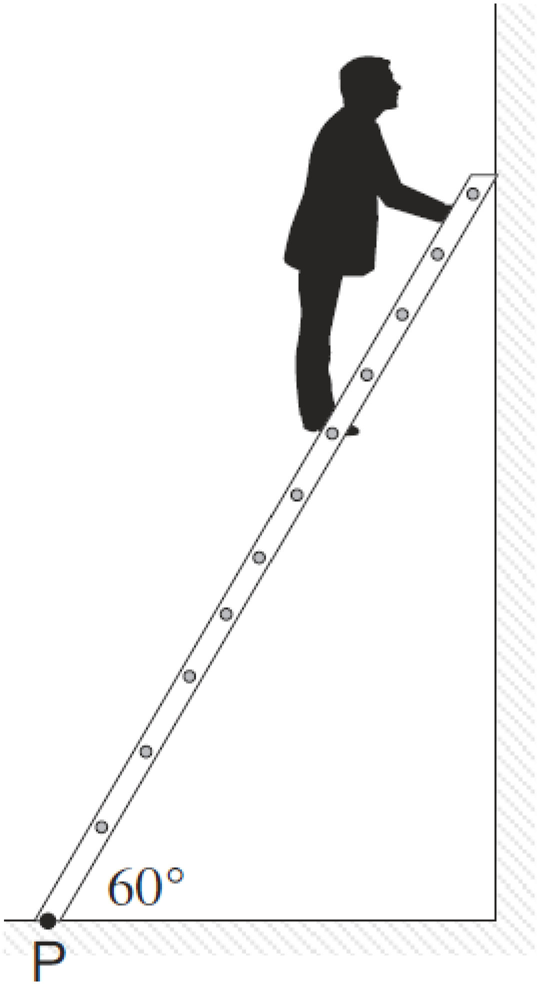


- τ_{muscle} = 19.2 N•m
- τ_{muscle} = 4.8 x 10 ^{2} N UP

Example

A 65 kg person is ¾ of the way up the 4.0 m ladder as shown in the diagram. What are the magnitude and direction of the torque about the base of the ladder at P produced by the person?

	Magnitude of torque	Direction
A	9.8×10^2 N•m	clockwise
B	9.8×10^2 N•m	counter clockwise
C	1.7×10^3 N•m	clockwise
D	1.7×10^3 N•m	counter clockwise



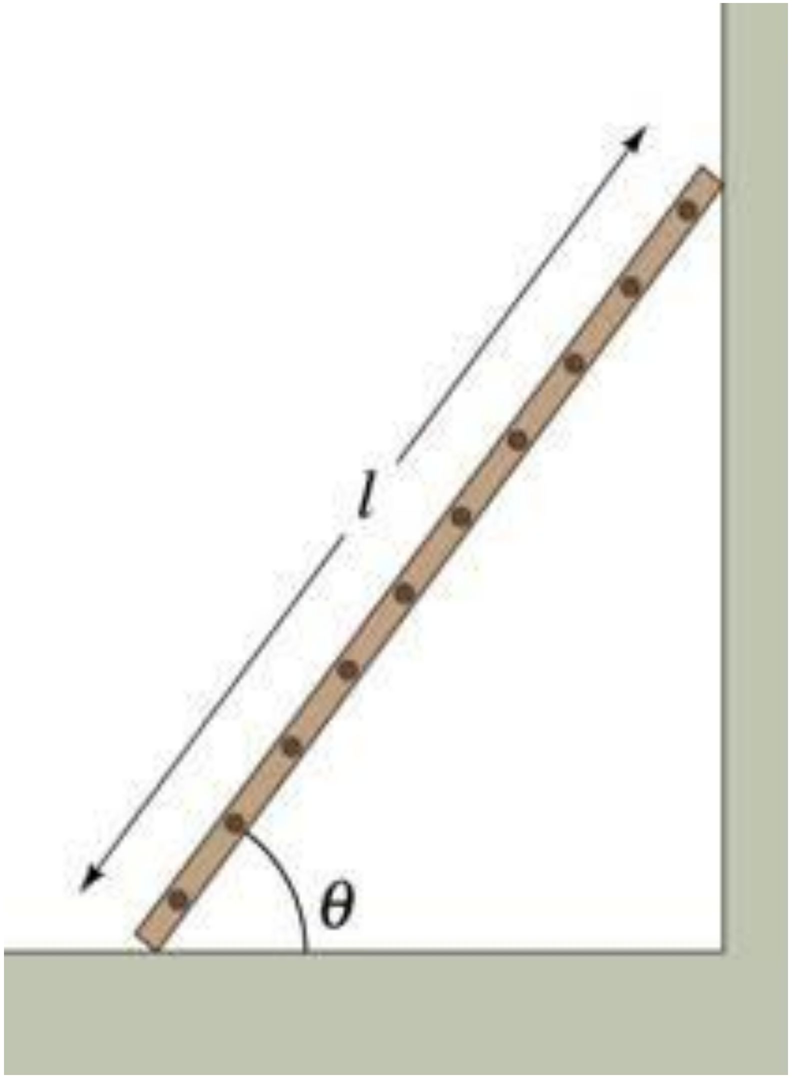
Solution

- $\tau = rF\sin\theta$

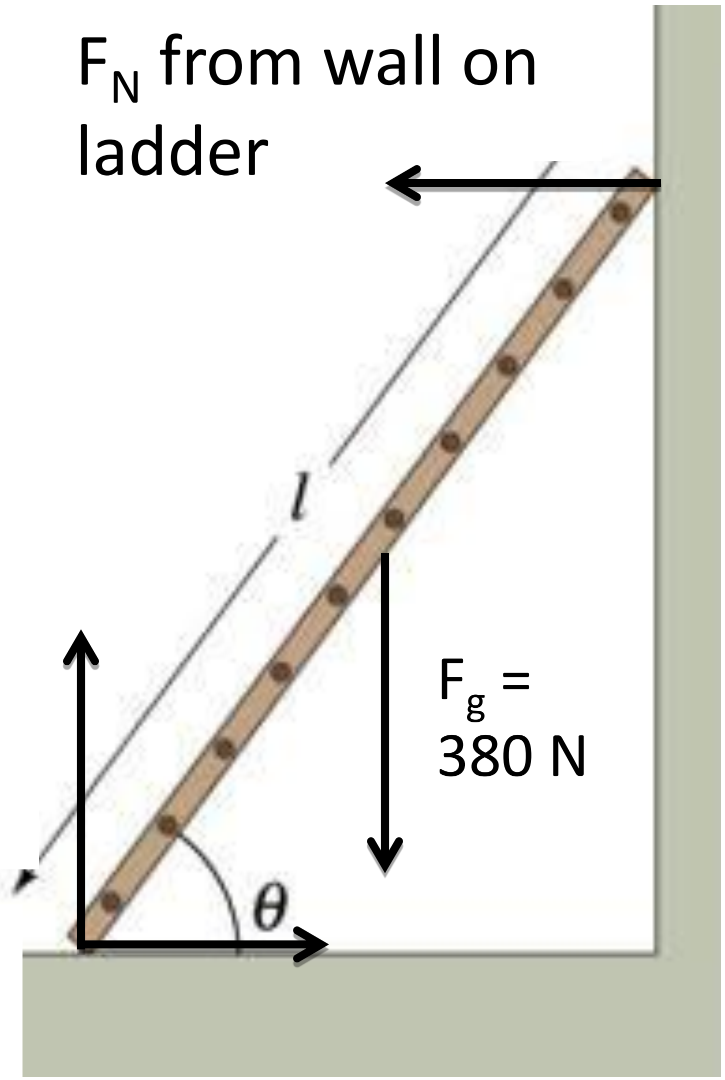
- τ = 3.0 m x 65 kg x 10 m/s^2 x sin 30^o
- τ = 3.0 m x 65 kg x 10 m/s^2 x ½
- τ = 3.0 m x 325 N
- τ = 975 N•m

Example

- A 4.00 m long ladder weighing 380 N is leaning against a wall. The ladder makes a 60^o angle with the ground and there is no friction between the ladder **<u>and the wall.</u>** Determine the force of friction between the ladder and the floor.



Solution
FBD!



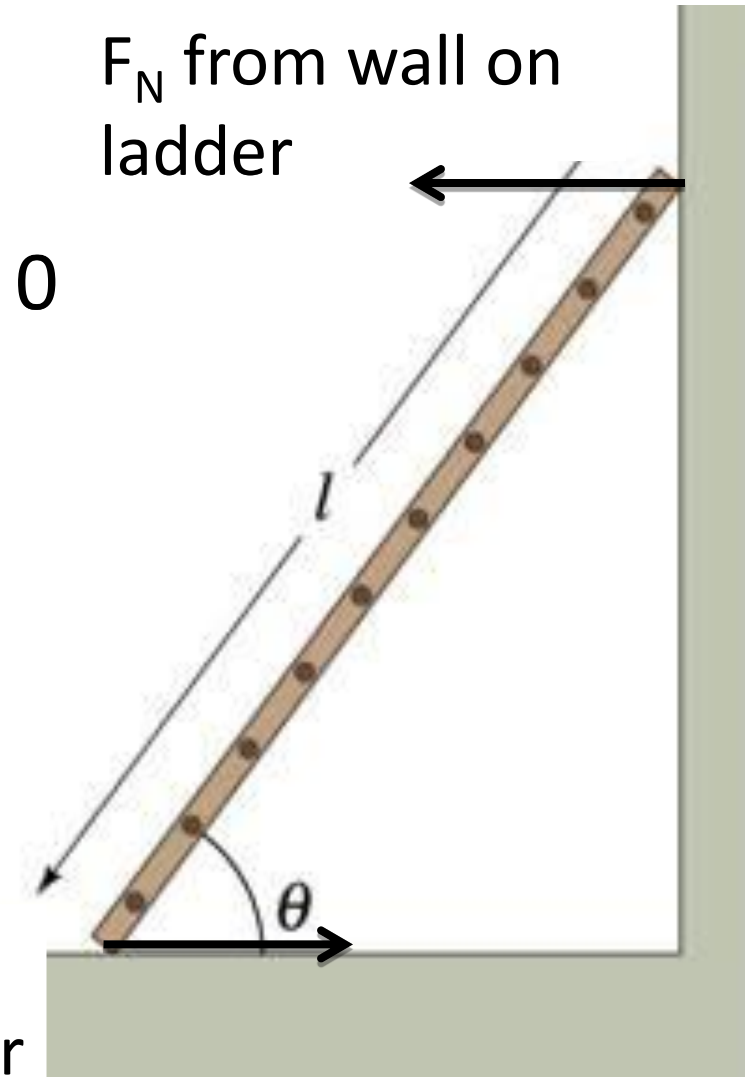
Solution
x-direction

F_{N} from wall on ladder

$\Sigma \mathbf{F} = 0$

$F_{N\ \text{from wall}} + F_{\text{friction from ground}} = 0$

$F_{N\ \text{from wall}} = - F_{\text{friction from ground}}$



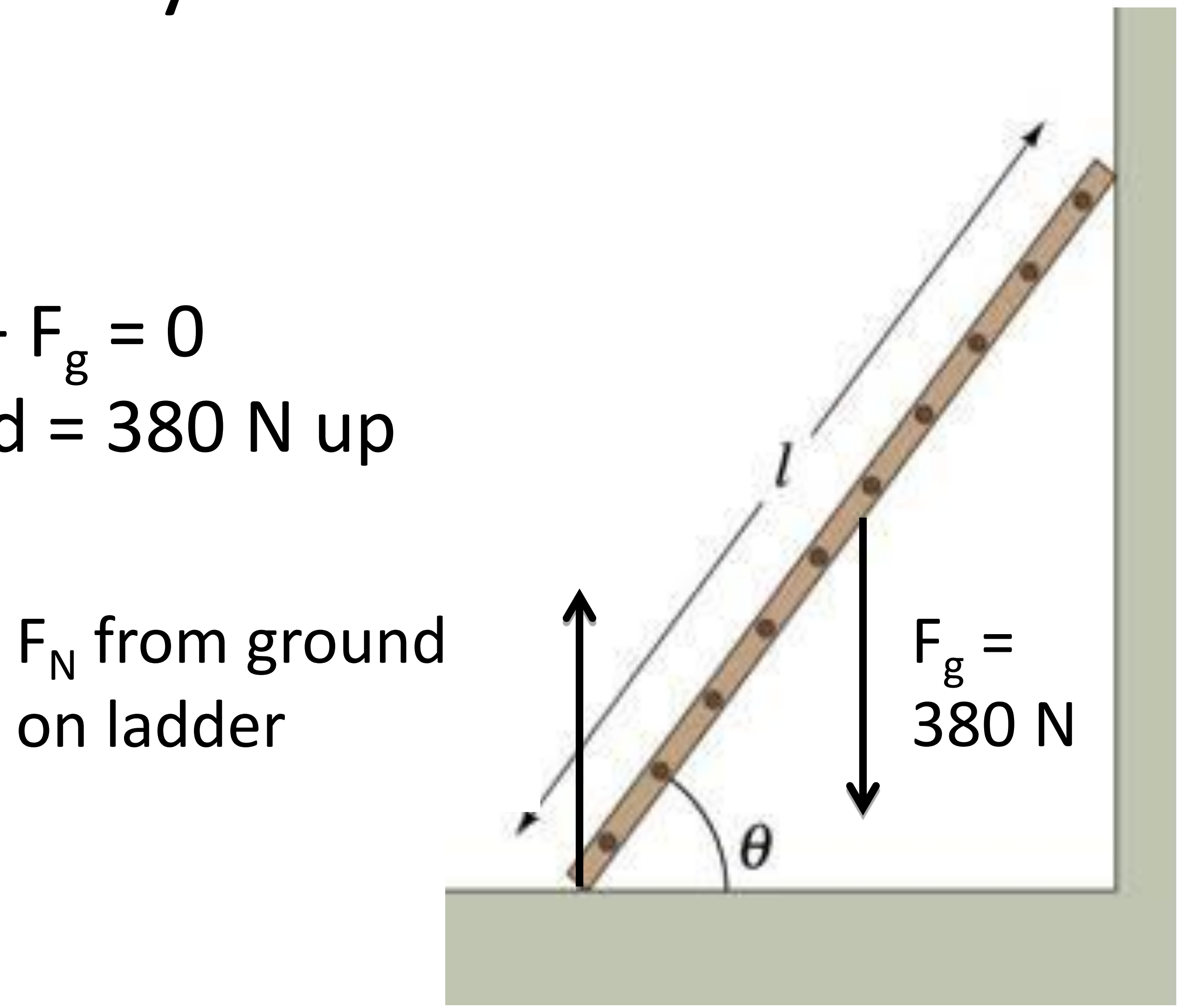
F_{friction} from ground on ladder

Solution
y-direction

$\Sigma \mathbf{F} = 0$

$F_{N\ from\ ground} + F_g = 0$

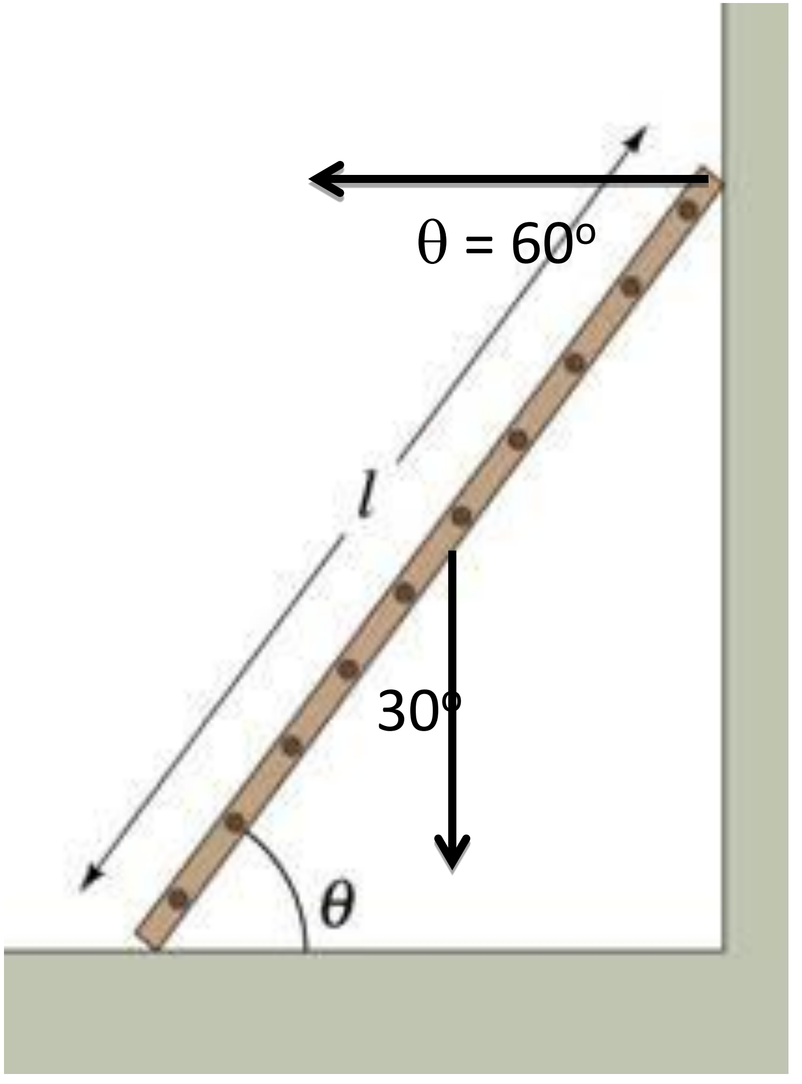
$F_{N\ from\ ground} = 380$ N up



Solution

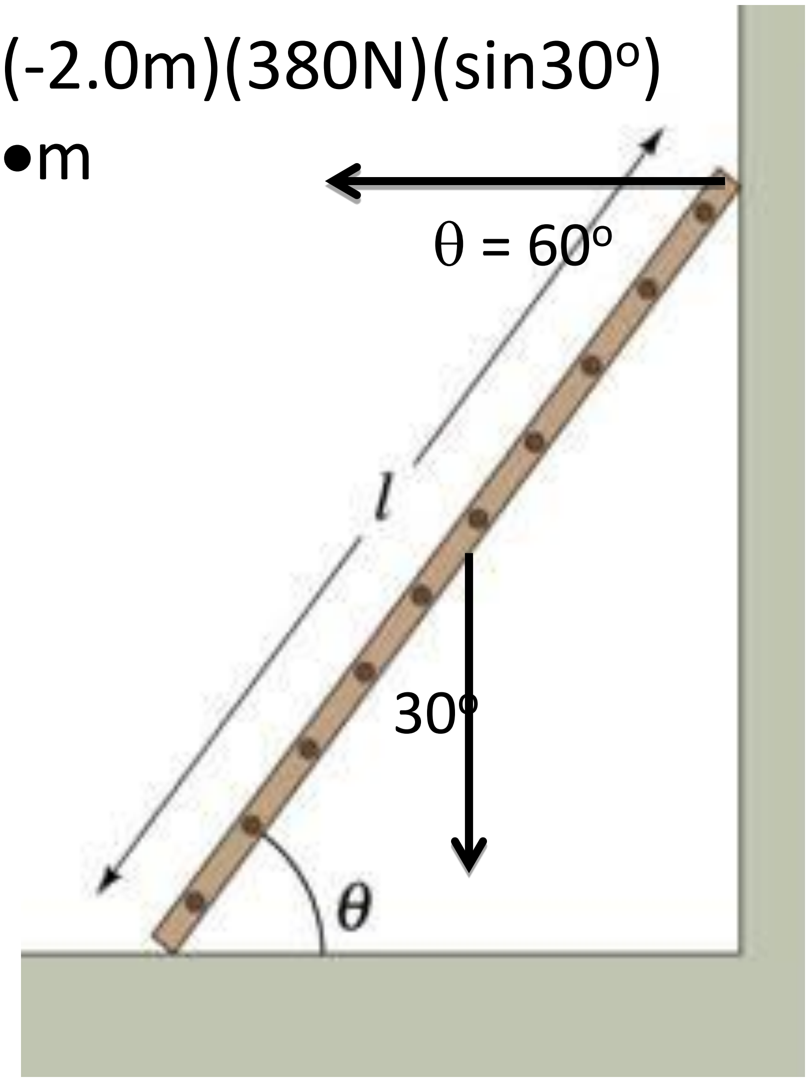
if the wall wasn't there, the ladder would pivot about the point of contact with the ground.

$\Sigma\tau = 0$



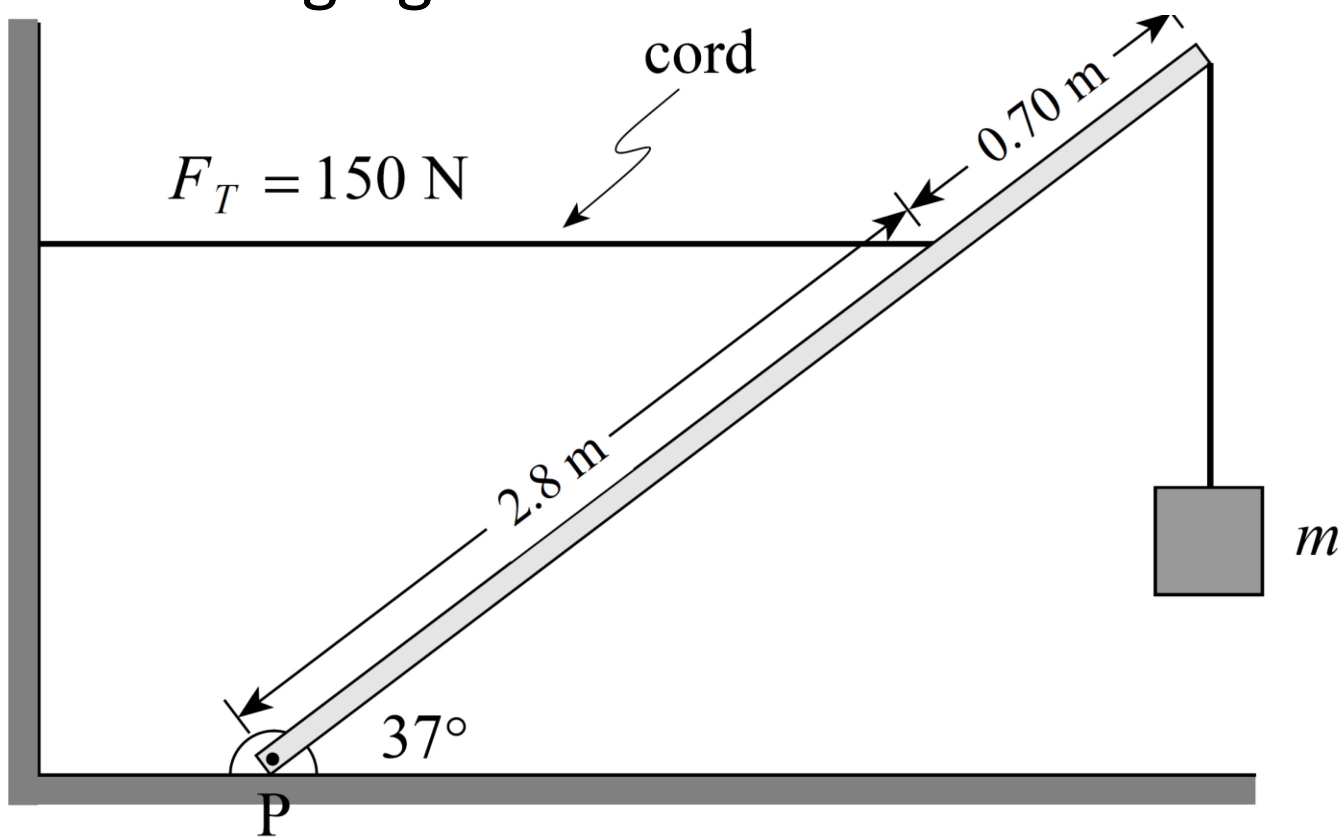
Solution

- $0 = (4.00\ m)(F_{N\ wall})(\sin 60^o)+(-2.0m)(380N)(\sin30^o)$
- $0 = 3.464\ m \times F_{N\ wall} + -380\ N\bullet m$
- $380\ N\bullet m = 3.464\ m \times F_{N\ wall}$
- $F_{N\ wall} = 110\ N$
- Since $F_{N\ wall} = F_{friction}$
- $F_{friction} = 110\ N$



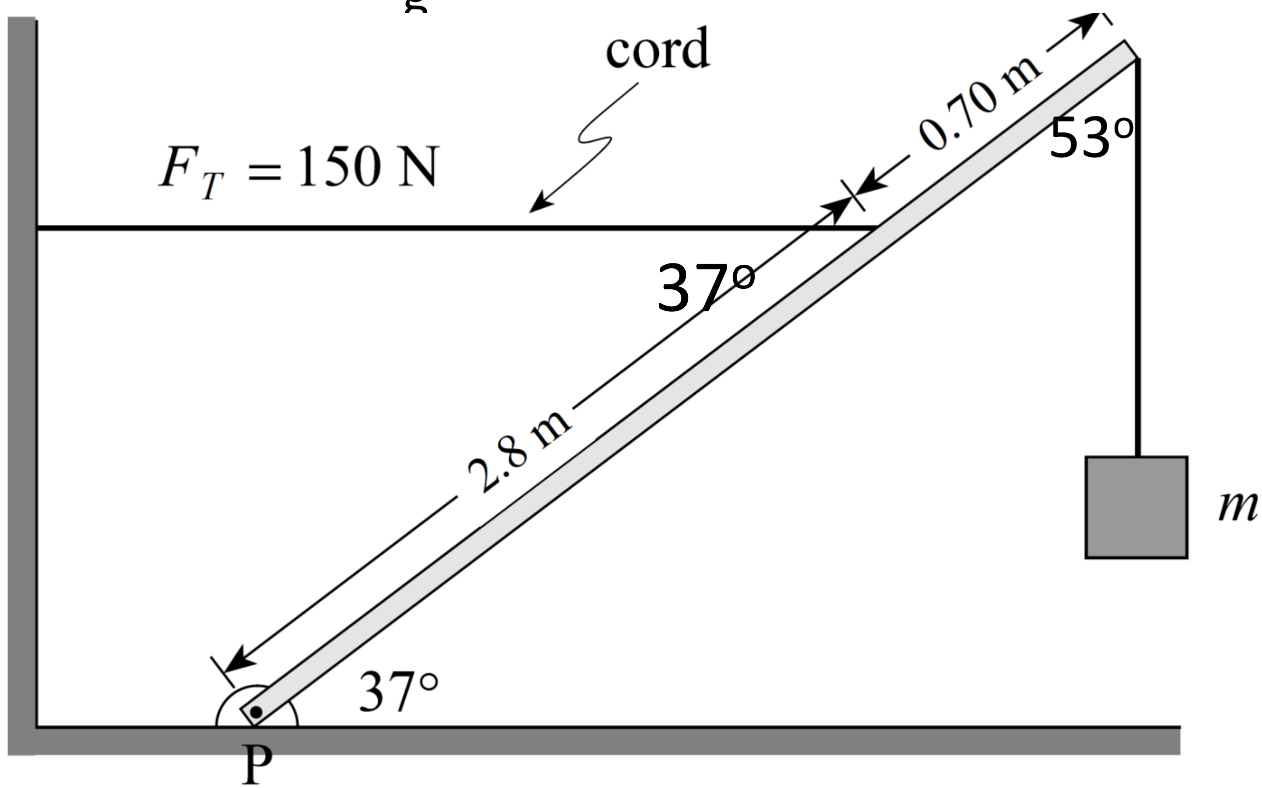
Example

- A uniform 3.5 m beam of negligible mass, hinged at P, supports a hanging block as shown. If the tension F_T in the horizontal cord is 150 N, what is the mass of the hanging block?



Solution

- $\Sigma\tau = 0$
- $\tau_{cord} + \tau_{mass} = 0$
- 2.8 m x 150 N x sin 37° + - 3.5 m x F_g x sin 53° = 0
- 252.76 N•m = 2.795 m x F_g
- F_g = 90.43 N
- m = 9.2 kg



Torque & Rotational Inertia

- A net force causes a mass to accelerate (translational motion)
- A net torque causes a mass to rotate, which is accelerated motion

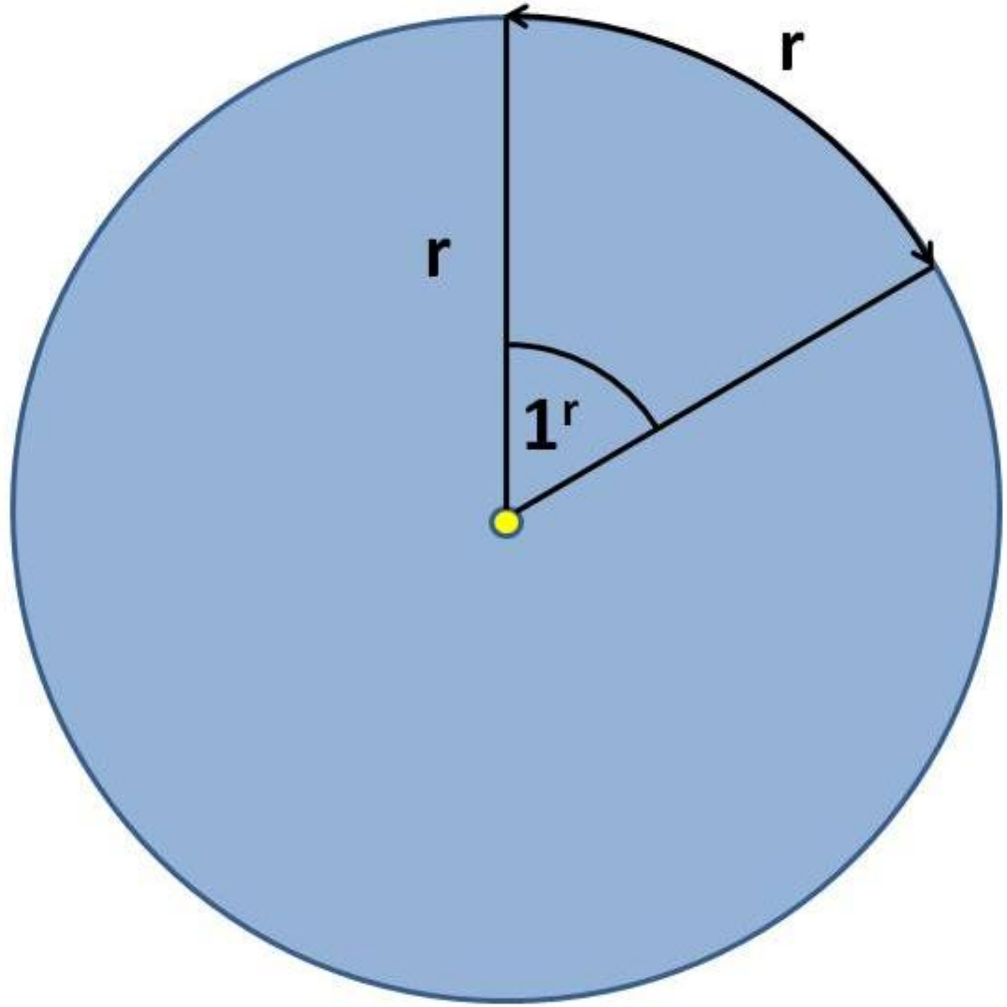
Angular Acceleration

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

- I is the moment of inertia or rotational inertia (units are kg•m^2)
- $I = \Sigma mr^2$
- Units of $\vec{\alpha}$ are radians/s^2

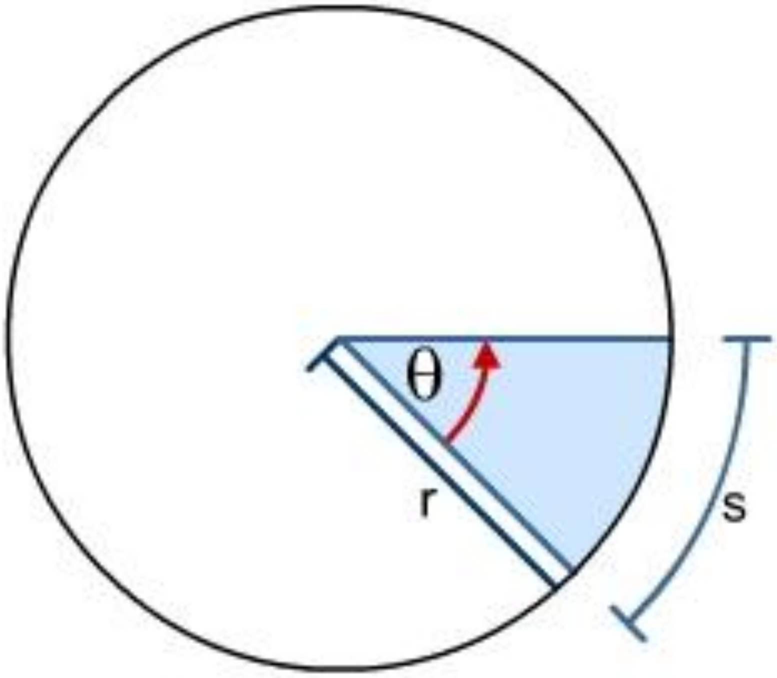
Definition of Radian

- A radian is the angle subtended by an arc that is equal to the radius of the circle



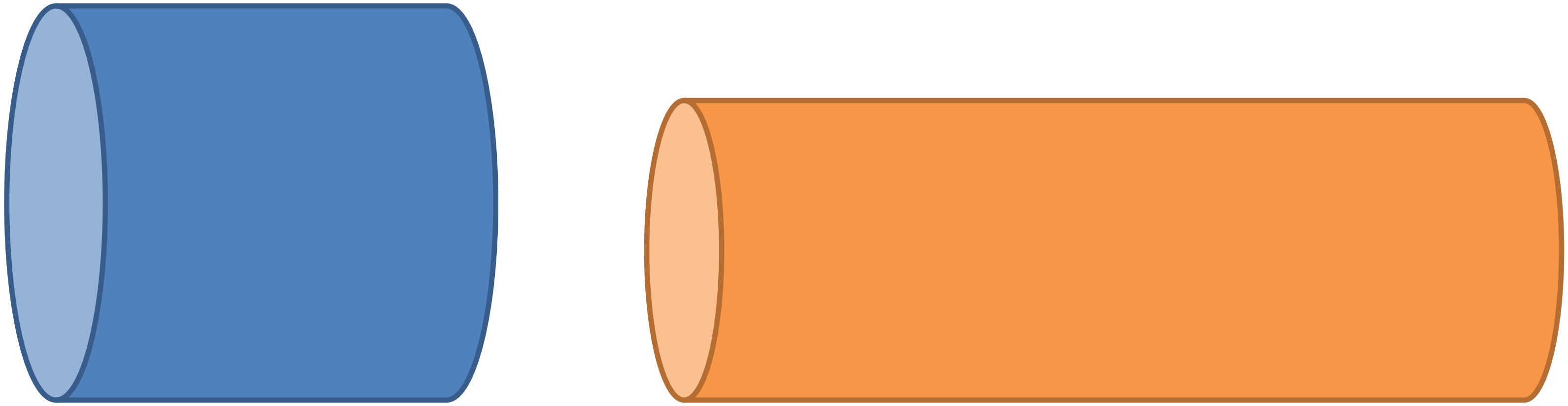
Arc length

- Length of arc, s, = θr



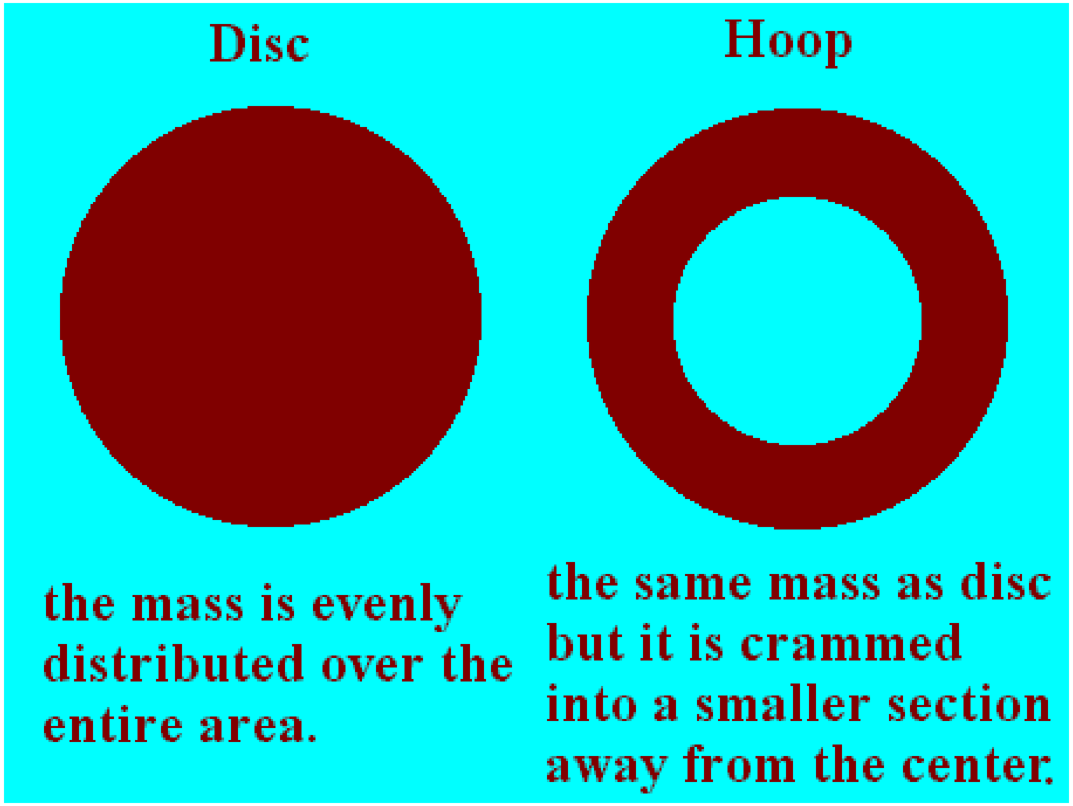
θ is measured in radians

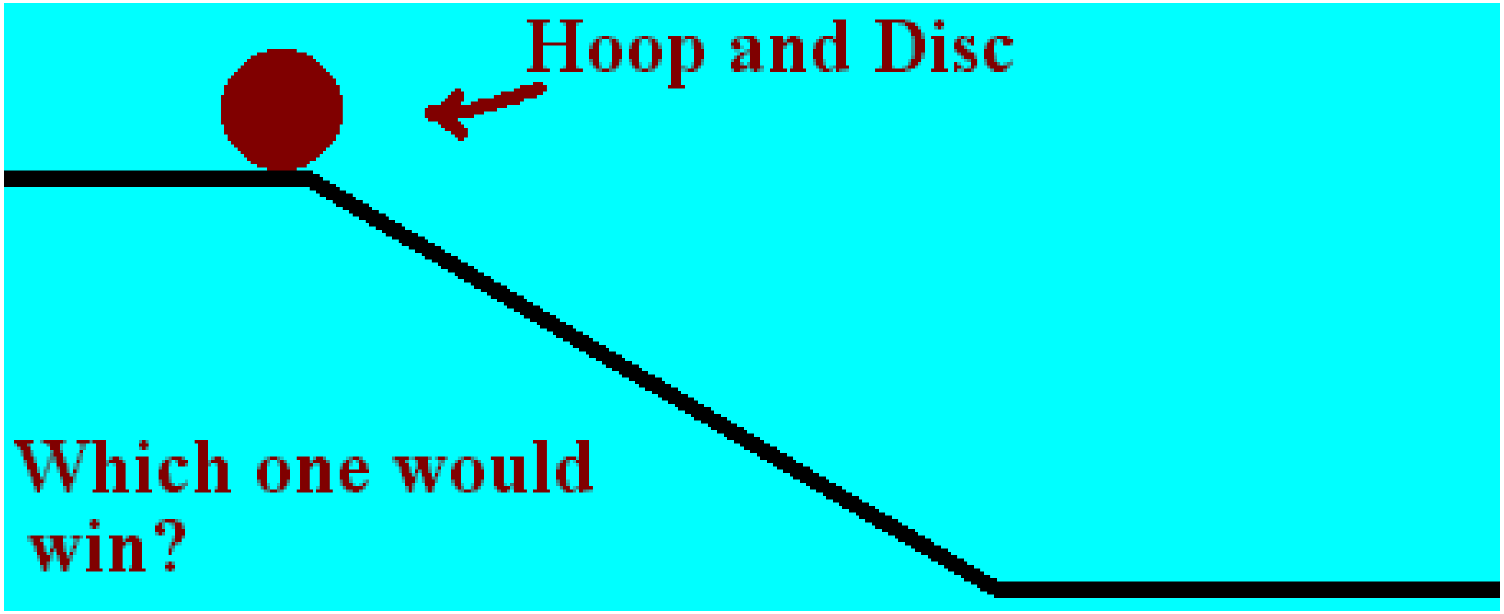
- Two cylinders have the same mass but different radii. Which one has the higher moment of inertia?



Example

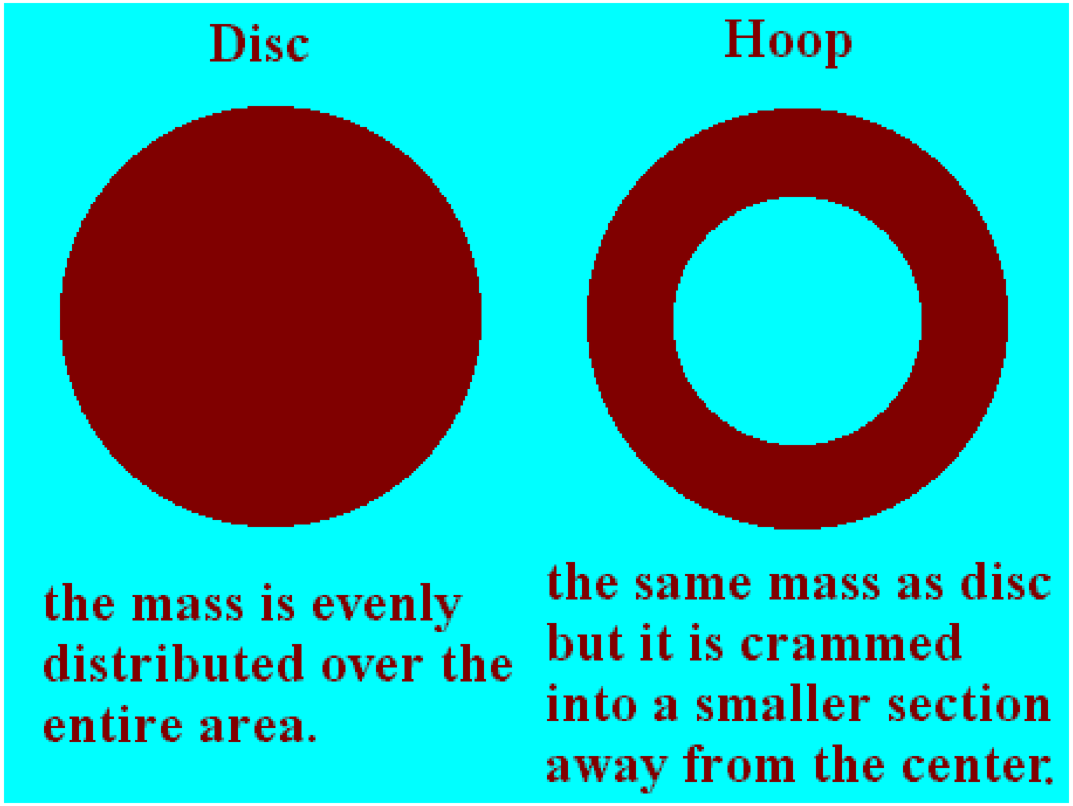
- Consider two objects: a hoop and a solid disc. If they start from the same position on an inclined ramp, which one would make it to the bottom of the ramp first?





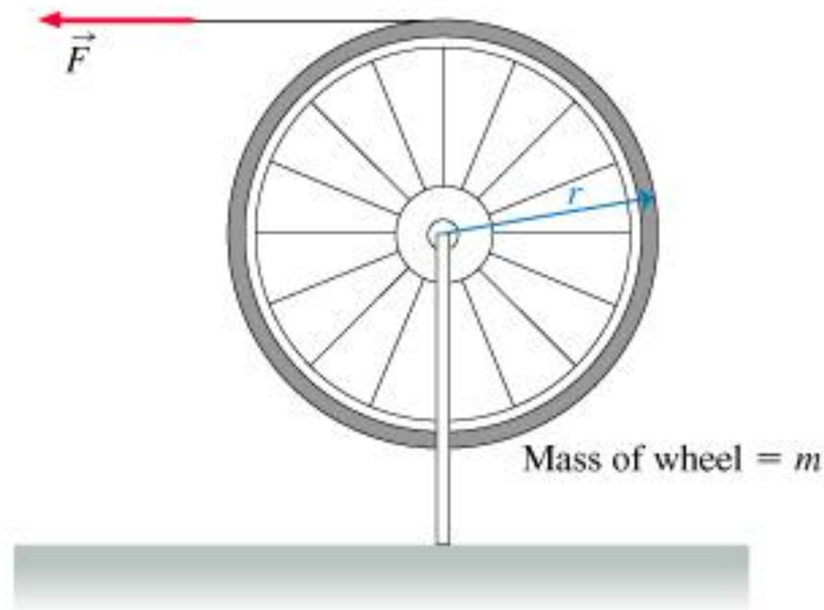
- the solid disc would beat the hoop to the bottom of the ramp!
- m**r**^{2} is the rotational inertia of the object: the disc has more of its mass at a greater radius
- ∴ more inertia

- $I_{disc} = \frac{1}{2} mr^2$
- $I_{hoop} = mr^2$



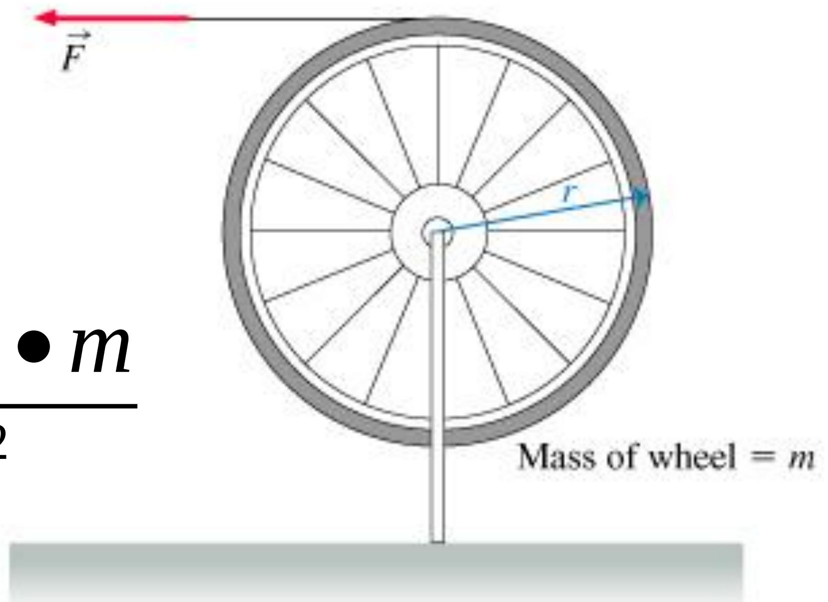
Example

- A 15 N force acts on a 4.0 kg wheel 33 cm in radius. There is a frictional torque of 1.1 N•m at the axle. Determine the angular acceleration of the wheel.



Example

- I = Σmr² = 0.4356 kg •m²



$$\vec{\alpha} = \frac{\Sigma \vec{\tau}}{I} = \frac{rF\sin\theta + -1.1N \bullet m}{0.4356 N \bullet m^2}$$

$$\vec{\alpha} = 8.8 \,{}^{rad}\!/_{s^2}$$