

# Momentum Conservation



During collisions

- in a collision, all the momentum before = all the momentum after
- Momentum is conserved

# Collision Types

- hit and stick, objects collide and remain together

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}'_{1 \text{ and } 2}$$

Mass object 1      Velocity of # 1 before      Mass object 2      Velocity of # 2 before      Mass object 1+ 2      Velocity of # 1 and 2 **AFTER collision**

The diagram shows the equation for a perfectly inelastic collision. On the left side, the sum of the initial momenta is shown as  $m_1 \vec{v}_1 + m_2 \vec{v}_2$ . Arrows point from labels to each term: 'Mass object 1' points to  $m_1$ , 'Velocity of # 1 before' points to  $\vec{v}_1$ , 'Mass object 2' points to  $m_2$ , and 'Velocity of # 2 before' points to  $\vec{v}_2$ . On the right side, the final momentum is shown as  $(m_1 + m_2) \vec{v}'_{1 \text{ and } 2}$ . An arrow points from the label 'Mass object 1+ 2' to the sum  $(m_1 + m_2)$ , and another arrow points from 'Velocity of # 1 and 2 AFTER collision' to  $\vec{v}'_{1 \text{ and } 2}$ .

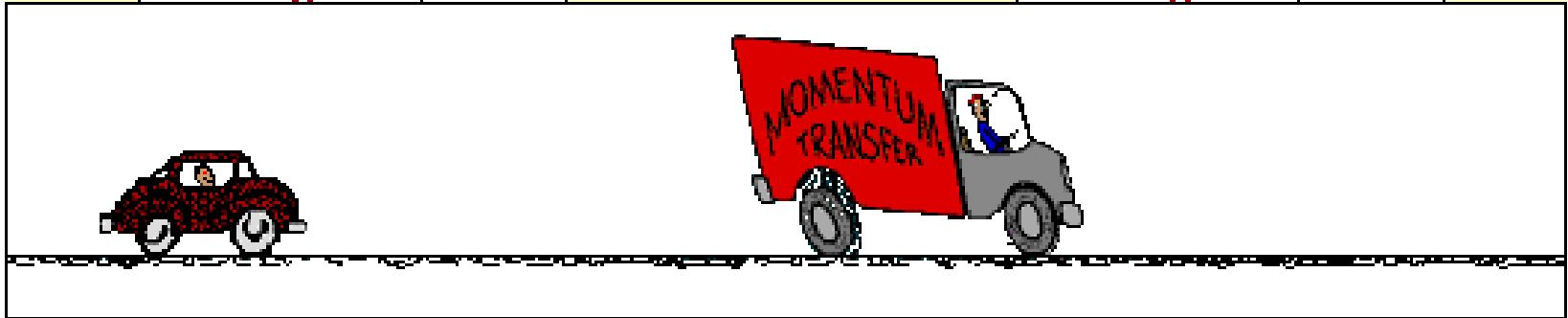
# Hit and Stick

Car

mass (kg)	1000
vel. (m/s)	20.0
mom. (kg m/s)	20 000

Truck

mass (kg)	3000
vel. (m/s)	0.0
mom. (kg m/s)	0



# Example

Chainsaw the cat ( $m = 2.50 \text{ kg}$ ) is running east at  $4.90 \text{ m/s}$  when it collides with Fluffy the poodle ( $m = 3.41 \text{ kg}$ ) walking west at  $4.90 \text{ m/s}$ . If the 2 remain together, determine their velocity after the collision.

# Solution

- $m_1 = 2.50 \text{ kg}$        $m_2 = 3.41 \text{ kg}$
- $v_1 = 4.90 \text{ m/s east}$        $v_2 = 1.00 \text{ m/s west}$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}'_{1 \text{ and } 2}$$

- $2.50(4.90) + 3.41(-1.00) = (2.50 + 3.41)v'_{1 \text{ and } 2}$
- $12.25 + -3.41 = 5.91v'_{1 \text{ and } 2}$
- $8.84 = 5.91v'_{1 \text{ and } 2}$
- $v'_{1 \text{ and } 2} = 1.50 \text{ m/s east}$

# Hit and bounce

- objects collide and do not remain together

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}'_1 + m_2 \vec{v}'_2$$

# Hit and Bounce



The momentum before a collision = momentum afterwards

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_1' + m_2 \vec{v}_2'$$

$$(800\text{kg})(+5.00\text{m/s}) + (900\text{kg})(-5.00\text{m/s}) = (800\text{kg})(-1.00\text{m/s}) + (900\text{kg})(+3.00\text{m/s})$$

$$-500 \text{ kg}\cdot\text{m/s} = -500 \text{ kg}\cdot\text{m/s}$$



# Explosions and Recoil

- 1 object breaks into 2 pieces

$$(m_1 + m_2) \vec{v}_{1 \text{ and } 2} = m_1 \vec{v}'_1 + m_2 \vec{v}'_2$$

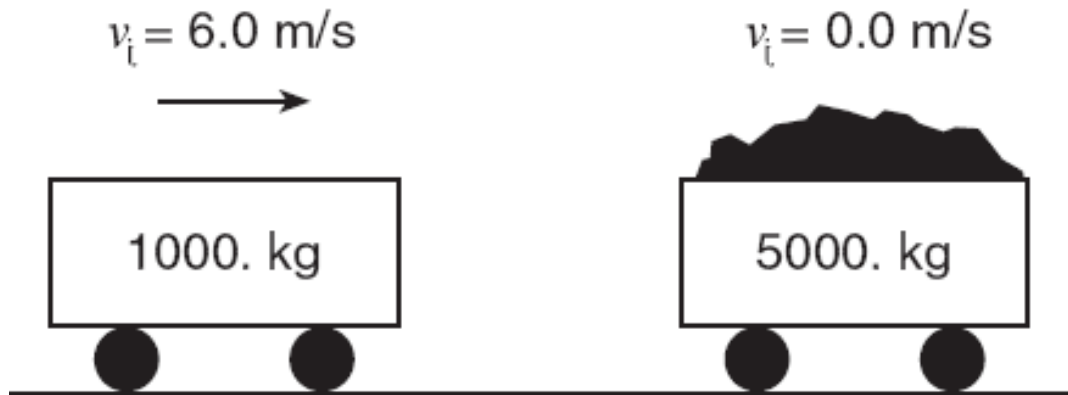
Diagram illustrating the conservation of momentum during an explosion. The equation shows the total mass before the explosion,  $(m_1 + m_2)$ , multiplied by the velocity before the explosion,  $\vec{v}_{1 \text{ and } 2}$ , equal to the sum of the mass and velocity of piece 1 after the explosion,  $m_1 \vec{v}'_1$ , and the mass and velocity of piece 2 after the explosion,  $m_2 \vec{v}'_2$ .

Labels for the equation components:

- Total mass before explosion (points to  $(m_1 + m_2)$ )
- Velocity before explosion (points to  $\vec{v}_{1 \text{ and } 2}$ )
- Mass and velocity of piece 1 **AFTER** (points to  $m_1 \vec{v}'_1$ )
- Mass and velocity of piece 2 **AFTER** (points to  $m_2 \vec{v}'_2$ )

# Example

- A 1000 kg railway car moving at 6.0 m/s to the right collides with a 5000 kg car at rest. If the two remain together, determine their final velocity. (Hit and stick collision)



$$\begin{aligned} m_1 &= 1000 \text{ kg} \\ v_1 &= 6.0 \text{ m/s right} \\ m_2 &= 5000 \text{ kg} \\ v_2 &= 0 \\ v'_{1+2} &= ? \end{aligned}$$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}'_{1 \text{ and } 2}$$

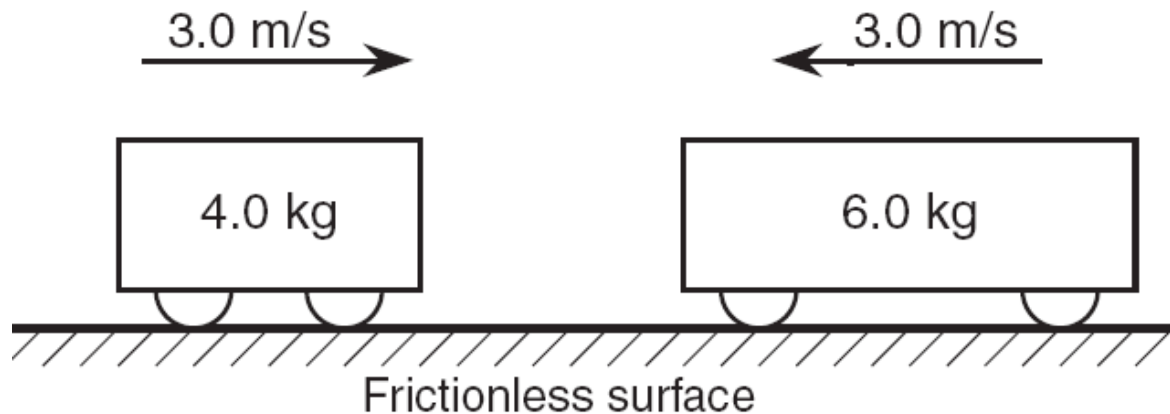
$$1000\text{kg}(6.0\text{m/s}) + 0 = (1000\text{kg} + 5000\text{kg}) \vec{v}'_{1 \text{ and } 2}$$

$$6000\text{kg} \cdot \text{m/s} = (6000\text{kg}) \vec{v}'_{1 \text{ and } 2}$$

$$\frac{6000\text{kg} \cdot \text{m/s}}{(6000\text{kg})} = \vec{v}'_{1 \text{ and } 2} = 1.0\text{m/s right}$$

# Example

- Two carts are moving as shown. When they collide they remain together. Determine their velocity after the collision.



**Watch the vector directions**

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}'_{1 \text{ and } 2}$$

Watch the vector directions

$$4.0\text{kg}(3.0\text{m/s}) + 6.0\text{kg}(-3.0\text{m/s}) = (4.0 + 6.0\text{kg}) \vec{v}'_{1 \text{ and } 2}$$

Right +                      left -

$$12.0\text{kg} \cdot \text{m/s} + -18\text{kg} \cdot \text{m/s} = (10\text{kg}) \vec{v}'_{1 \text{ and } 2}$$

$$\frac{-6.0\text{kg} \cdot \text{m/s}}{(10\text{kg})} = \vec{v}'_{1 \text{ and } 2} = -0.60\text{m/s}$$

$$\vec{v}'_{1 \text{ and } 2} = 0.60 \text{ m/s left}$$

# Example



The two objects shown above collide head-on. The velocity of the 9.5 kg object after collision is 5.4 m/s to the left.

- Determine the velocity of the 2.4 kg mass after the collision.
- Hit and bounce, **watch the vector directions**

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}'_1 + m_2 \vec{v}'_2$$

$$2.4\text{kg}(3.2\text{m/s}) + 9.5\text{kg}(-8.4\text{m/s}) = 2.4\text{kg} \vec{v}'_1 + 9.5\text{kg}(-5.4\text{m/s})$$

$$7.68\text{kg} \cdot \text{m/s} + -79.8\text{kg} \cdot \text{m/s} = 2.4\text{kg} \vec{v}'_1 + -51.3\text{kg} \cdot \text{m/s}$$

$$-72.12\text{kg} \cdot \text{m/s} = 2.4\text{kg} \vec{v}'_1 + -51.3\text{kg} \cdot \text{m/s}$$

$$-72.12\text{kg} \cdot \text{m/s} + 51.3\text{kg} \cdot \text{m/s} = 2.4\text{kg}$$

$$-20.82\text{kg} \cdot \text{m/s} = 2.4\text{kg} \vec{v}'_1$$

$$\frac{-20.82\text{kg} \cdot \text{m/s}}{2.4\text{kg}} = \vec{v}'_1 = -8.675\text{m/s} \quad v'_1 = 8.7 \text{ m/s left}$$

# Explosions & Recoil

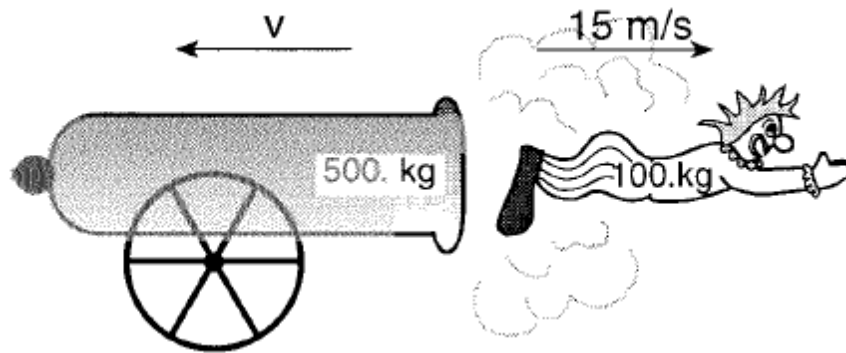






# Example

- A 100 kg clown is fired from a 500 kg cannon at rest. If the clown's velocity is 15 m/s afterwards, determine the recoil velocity of the cannon.



$$V_{1 \text{ and } 2} = 0 \text{ m/s}$$

$$m_1 = 100 \text{ kg}$$

$$m_2 = 500 \text{ kg}$$

$$v'_1 = 15 \text{ m/s forward}$$

$$v_{1 \text{ and } 2} = 0 \text{ m/s}$$

$$m_1 = 100 \text{ kg}$$

$$m_2 = 500 \text{ kg}$$

$$v'_1 = 15 \text{ m/s forward}$$

$$(m_1 + m_2) \vec{v}_{1 \text{ and } 2} = m_1 \vec{v}'_1 + m_2 \vec{v}'_2$$

$$(100 + 500) \times 0 = 100(15) + 500 \vec{v}'_2$$

$$0 = 1500 + 500 \vec{v}'_2$$

$$-1500 = 500 \vec{v}'_2$$

$$\frac{-1500}{500} = \vec{v}'_2 = -3.00 \text{ m/s}$$

the cannon moves backwards at 3.00 m/s



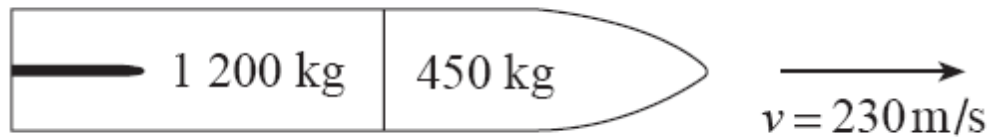
## Rescue Humor

The fun side of Police  
Fire and EMS

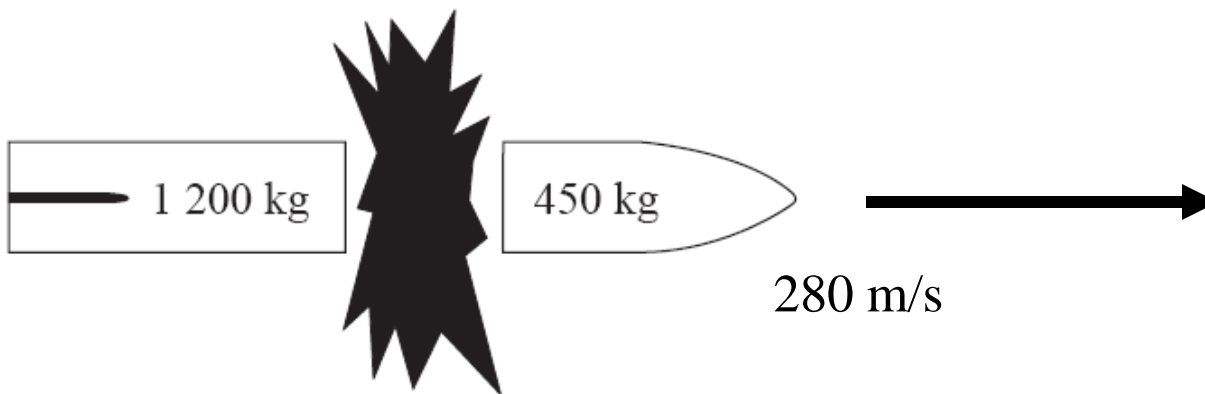


# Example

A space vehicle made up of two parts is travelling at 230 m/s as shown.



An explosion causes the 450 kg part to separate and travel with a final velocity of 280 m/s as shown.



- Determine the velocity of the 1200 kg part after the explosion.
- Watch the vector directions

$$(m_1 + m_2) \vec{v}_{1 \text{ and } 2} = m_1 \vec{v}'_1 + m_2 \vec{v}'_2$$

$$(1200 + 450) \times 230 = 450(280) + 1200 \vec{v}'_2$$

$$379500 = 126000 + 1200 \vec{v}'_2$$

$$253500 = 1200 \vec{v}'_2$$

$$\frac{253500}{1200} = \vec{v}'_2 = 211 \text{ m/s}$$

What direction is positive?

211 m/s forward (or to the right)



# Practice

- P 271: #4, 5, 6, 7, 8